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BUBBLE DISTRIBUTIONS IN THE UPPER OCEAN

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Monterey, California

THESIS

BUBBLE DISTRIBUTIONS IN THE UPPER OCEAN

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Bubble Distributions in the Upper Ocean

by

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ABSTRACT

A bubble transport equation has been developed that the bubble distribution, as a function of position, velocity, and size, must satisfy. An analytical model for bubble transport in the upper ocean is chosen and solutions are developed for this model. For a surface source only, these solutions compare favorably to experimental data in the near-surface region. The depth of this region of agreement depends upon the circulation field chosen, but is of the order of 3 meters.

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TABLE OF SYMBOLS

SYMBOL	DESCRIPTION
z	depth ($z \leq 0$)
$\bar{v}$	bubble velocity
$\bar{V}$	circulation field velocity
l	bubble radius
R	dl/dt
Ψ	bubble distribution (function of $\bar{v},\bar{r},l$)
S	distributive source
$\mathbf{V}$	bubble volume
ρ_w	density of water
ρ	density of gas in bubble
η	viscosity of water
$\bar{u}$	relative velocity ($\bar{v}-\bar{V}$)
$\bar{a}$	bubble acceleration
σ	fractional part of bubble volume
α	$9/2\ \eta/\sigma\rho_w$
P_o	atmospheric pressure
P	hydrostatic pressure
g	gravity
ϕ	bubble distribution (function of z,x,v_z,v_x,l)
α_1	$P_o/\rho wg$
Φ	bubble density (as a function of x,z,l), $\int \Psi d^3v$
$\bar{r}$	position

I. INTRODUCTION

Studies concerning the propagation of sound in the ocean and under water sound scattering from the sea surface have shown a discrepancy between existing theories and experimental results. Because air has a markedly different density and compressibility than sea water and because of the resonant characteristics of bubbles, the suspended air bubbles in sea water have a profound effect upon underwater sound and can explain part of the discrepancies that have been encountered (Ref. 1 and 2). There are theories that predict the effects of air bubbles on absorption, scattering and attenuation of sound as well as their effects on sound velocity (Ref. 3), but there is no theory at present which predicts the distribution of air bubbles in the ocean. Such a theory would enhance existing theories and result in better predictions on the behavior of sound propagation in the upper ocean.

A few experimental studies have been conducted on the distribution of bubbles in the ocean. Blanchard and Woodcock were pioneers in this field, using a bottle to scoop up water from breaking waves and counting the number of bubbles of different sizes (Ref. 4). Later studies conducted at the U.S. Naval Postgraduate School, Monterey, used acoustic techniques to determine the bubble population, taking advantage of the fact resonant acoustical scattering

and absorption cross section of bubbles are of the order of a thousand times the geometrical cross section. These experiments indicated that bubbles of radius 60 microns or greater have a distribution depth dependence of $Z^{-\frac{1}{2}}$, whereas smaller bubbles follow closer the exponental law $e^{-z/L}$, where L is between 5 and 9 meters (Ref. 5).

In this paper, development of a bubble transport equation has been outlined that the bubble distribution, as a function of position, velocity and size, must satisfy. An analytical model for bubble transport in the upper ocean is chosen and solutions are developed for this model.

II. TRANSPORT EQUATION

A heuristic approach is used here to develop the bubble transport equation, since the results are the same as with a more rigorous approach and the mathematics is less involved (Ref. 12). Consider the distribution $\Psi(\bar{r},\bar{v},l,t)d^3rd^3vdl$ which defines the number of bubbles with radii in dl about l and with velocity in d^3v about v which are in a volume d^3r about r at time t. If collisions among the bubbles are neglected, then the following equations can be written for the seven-dimensional space:

$$\frac{\partial\Psi}{\partial t}(\bar{r},\bar{v},l,t)d^3rd^3vdl = \left[\text{net flux in} + S(\bar{r},\bar{v},l,t)\right] d^3rd^3vdl \quad (1)$$

where $S(\bar{r},\bar{v},l,t)d^3rd^3vdl$ is a distributed source of bubbles introduced in d^3r about $\bar{r}$ at time t with radii in dl about l and velocity in d^3v about $\bar{v}$.

Referring to figure 1, which represents a volume element in real space, the net flux in, per unit volume, for the x-direction, is:

$$\frac{\text{fluxin-fluxout}}{\text{volume}} = \frac{v_x(x)\Psi(x)dydz - v_x(x+dx)\Psi(x+dx)dydz}{dxdydz} = -\frac{\partial(v_x\Psi)}{\partial x}$$

Similiarly, the net flux in per unit volume for the y and z-directions is $-\frac{\partial(v_u\Psi)}{\partial y}$ and $-\frac{\partial(v_z\Psi)}{\partial z}$, respectively. Combining the above results, the total net flux in per unit volume for real space is $-\nabla_r\cdot(\bar{v}\Psi)$, where the subscript r on the del-operator means the operation is taking place in the real space. A similiar approach is used to obtain the total net flux in for velocity and size space: $-\nabla_v\cdot(\bar{a}\Psi)$

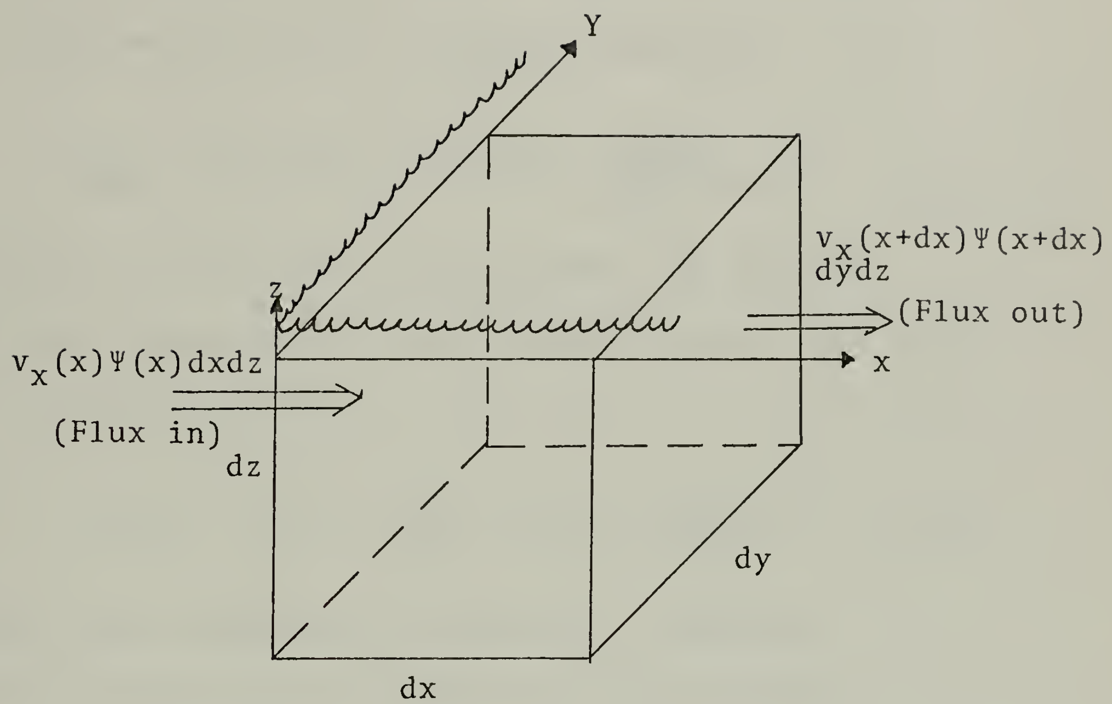

Figure 1

Elemental Volume Showing "Bubble" Streaming in Real Space (x,y,z).

for velocity space and $-\frac{\partial(R\Psi)}{\partial l}$ for size space, where $R \equiv dl/dt$. The total net flux in for the seven-dimensional space under consideration is the sum of the net flux in for real, velocity, and size spaces. Substituting this into equation (1) gives the general bubble transport equation:

$$\frac{\partial\Psi}{\partial t} = -\nabla_r \cdot (\bar{v}\Psi) - \nabla_v \cdot (\bar{a}\Psi) - \frac{\partial(R\Psi)}{\partial l} + S \qquad (2)$$

The remainder of this paper will consider the special case of steady-state with no distributed sources, for which equation (2) can be written

$$\bar{v} . \nabla_r \Psi + \bar{a} . \nabla_v \Psi + \frac{R\partial\Psi}{\partial l} = -\left[\nabla_v . \bar{a} + \frac{\partial R}{\partial l}\right] \Psi \qquad (3)$$

since $\bar{r}$ and $\bar{v}$ are independent variables. Although distributed sources will be neglected in this thesis, surface sources can be introduced as boundary conditions. It should be pointed out that the non-zero right hand side of equation (3) is peculiar to the bubble transport problem and arises becouse $\bar{a}$ is a function of $\bar{v}$ and R is a function of l.

III. ANALYTICAL MODEL FOR TRANSPORT EQUATION

Before solutions to equation (3) can be obtained, a realistic model for the bubble transport equation must be developed, with particular attention to formulating expressions for the bubble acceleration ($\bar{a}$), the time rate of change of the bubble radius ($\dot{R}$) and the circulation field ($\bar{V}$).

A. MODEL ASSUMPTIONS

The model used in this development assumes Reynold's numbers less than one, which is the Stokes' Law Regime. With regard to this assumption two comments should be made. First, for small Reynold's numbers, bubbles will behave like solid spheres with little or no distortion and secondly, this places an upper limit of about 150 microns on the size of bubbles that can be considered. For Reynold's numbers much greater than one, (larger bubbles) the resistance of the turbulent wake will have to be included in the equation of motion, along with Stokes' drag force (Ref. 6).

Surface tension has been neglected in the model, so the pressure inside the bubble is equal to the hydrostatic fluid pressure. Since the pressure from surface tension $2\gamma/1$, where γ is approximately $74\text{x}10^{-3}$ NT/m for sea water, this assumption will introduce an error of less than 5 percent for radii greater than 30 microns. Thus, where the

Reynold's number places an upper bound on the radius, neglecting surface tension places a lower bound on the bubble radius under consideration.

The volume of a gas bubble in a liquid varies primarily through the effect of two influences. First, gas continuously leaks out of the bubble to dissolve in the surrounding liquid. If the bubble was stationary, the ensuing loss of mass would necessarily lead to its ultimate disappearance. On the other hand, as a bubble assends or desends in the liquid, the ambient hydrostatic pressure decreases or increases and so does the pressure within the gas bubble, thus effecting the size.

The effects of the interaction between gas diffusion and change in hydrostatic pressure can best be seen by looking at the ideal gas law:

$$\frac{4}{3}\pi l^3 = rTN/P$$

where N is the total number of gas molecules, r is the gas constant, T is the temperature, and P is the pressure. Now, taking the time derivative of the ideal gas law equation and assuming isothermal changes,

$$4\pi l^2 \dot{R} = \frac{rT}{P^2} \; PdN/dt - Ndp/dt$$

and rearranging:

$$R = \frac{1}{3} \; \frac{dN/dt}{N} - \frac{dP/dt}{P}$$

The above development assumes a single gas inside the bubble vice some mixture of gases. In equation (4), the first term is a result of gas diffusion whereas the second term brings in the effects of compression.

Due to the relative motion between the water and bubble, assume that the composition of the water in respect to dissolved gas is everywhere uniform and saturated at atmospheric pressure, P_o, except in a thin shell surrounding the bubble. Assume that the gas in the bubble is uniform right up to the gas-water interface, and that the liquid in contact with the bubble is saturated at the pressure, P, inside the bubble. Further assume that diffusion gradient is uniform throughout the thin shell. With these assumptions, Fick's First Law can be used, which relates the amount of material which diffuses through an unit area to the product of the concentration gradient and diffusion constant:

$$dN/dt = -\delta 4\pi l^2 (P-P_o).$$

In this expression δ is a proportionality constant which depends on d, the thickness of the shell; on Δ, the diffusion constant of the gas in the water; and on α, the solubility of the gas in the water, in accordance with the relation:

$$\delta = \Delta\alpha/d.$$

Using Fick's First Law with the hydrostatic pressure relationship, $P=P_o - \rho_w gz$, equation (4) becomes:

$$R = \delta rT \frac{z}{(\alpha_1 - z)} + \frac{1}{3}\frac{v_z}{(\alpha_1 - z)} \qquad (4)$$

where $\alpha_1 \equiv P_o/\rho_W g$ and v_z is the bubble velocity in the z-direction. In reference 7, the expression δrT was determined experimentally to be approximately a constant, 10^{-6} m/sec.

B. EQUATION OF MOTION

When air bubbles are entrained in a circulation field, there are three forces acting on them along with the bouyant force. Figure 2 illustrates an air bubble in a circulation field and shows graphicly the various forces which need to be considered in order to determine the bubble acceleration.

If the bubbles were completely entrained by the fluid, i.e., bubble velocity = circulation field velocity, they would experience the same force as would the fluid enclosed in the same volume, $\rho_w V\, d\bar{V}/dt$. However, complete entrainment can only occur if $\rho = \rho_W$, which is far from being true for air bubbles. Since the bubbles are only partially entrained, the fluid flows past the bubbles creating a relative velocity ($\bar{u}$) and thus introduces a drag force. As mentioned earlier, small Reynold's numbers have been assumed and thus the drag acting on the bubble can be described by Stokes' Law,

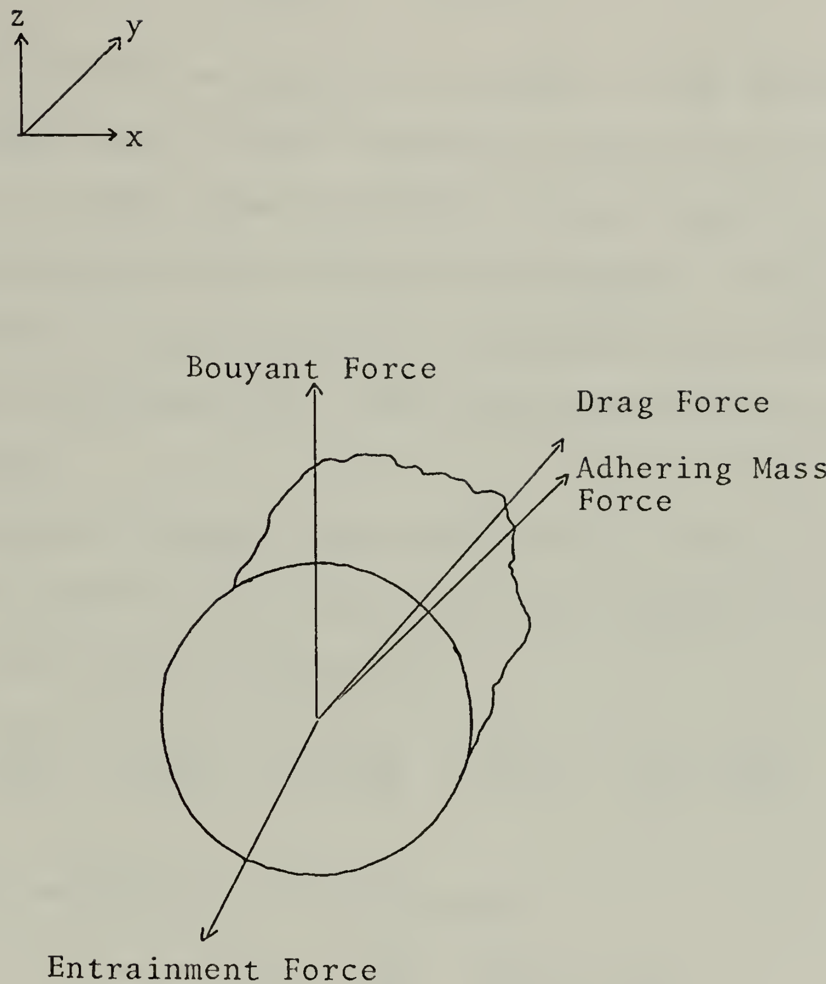

Figure 2

Free Body Diagram of a Bubble,
Showing the Forces Acting on
the Bubble in a Circulation Field

$$\bar{F}_{Drag} = - 6\pi\eta l\bar{u}.$$

The third force acting on the bubble is the innertial adhering mass force. When a spherical shape moves through the water, it drags along an amount of water equal to a fractional amount of the volume of the sphere. In the equations that follows, sigma (σ) represents this fractional part, and in reference (8), sigma has been derived as 0.5 for a sphere.

Referring to figure 2 and using Newton's Second Law of Motion, the equation of motion for an air bubble in a circulation field can be written as:

$$\rho\mathcal{V}\frac{d\bar{v}}{dt} = \rho_w\mathcal{V}\frac{d\bar{V}}{dt} - \sigma\mathcal{V}\rho_w\frac{d\bar{u}}{dt} - 6\pi\eta l\bar{u} + (\rho_w-\rho)\mathcal{V}g\hat{k}.$$

Using $\rho<<\rho_w$ and $d\bar{V}/dt = (\bar{v}\cdot\nabla_r)\bar{V}$ yields

$$a = - \frac{\alpha}{l^2}\bar{u} + \frac{g\hat{k}}{\sigma} + \frac{1+\sigma}{\sigma}(\bar{v}\cdot\nabla r)\bar{V} \qquad (5)$$

where $\alpha \equiv 9/2\ n/\sigma\rho_w$ and $\bar{u}=\bar{v}-\bar{V}$.

C. CIRCULATION FIELD

Little is known about the circulation field in the upper ocean. Although some work is being conducted at present at the U.S. Naval Postgraduate School, Monterey. In reference (6), Levich develops a two-dimensional equation which describes the particle motion by wave action in an ideal liquid, and is the basic form used here to describe circulation field:

$$\bar{V} = V_o e^{+kz} \sin kx\hat{i} - V_o e^{+kz} \cos kx\hat{k}. \qquad (6)$$

This equation gives a periodic form for the horizontal direction as well as an exponental decay in depth, which might be expected for the physical situation in the upper ocean. It is also well to note that the condition for an imcompressible fluid has been satisfied with this equation, i.e., $\nabla \cdot \bar{V} = 0$.

Figure 3 shows the cell structure implied by this circulation field. For this model, any two adjacent cells are identical to every other pair of adjacent cells that are separated from the first pair by an integer number of wavelengths. This implies that the boundary conditions must be periodic unless unphysical surface sources are to be assumed.

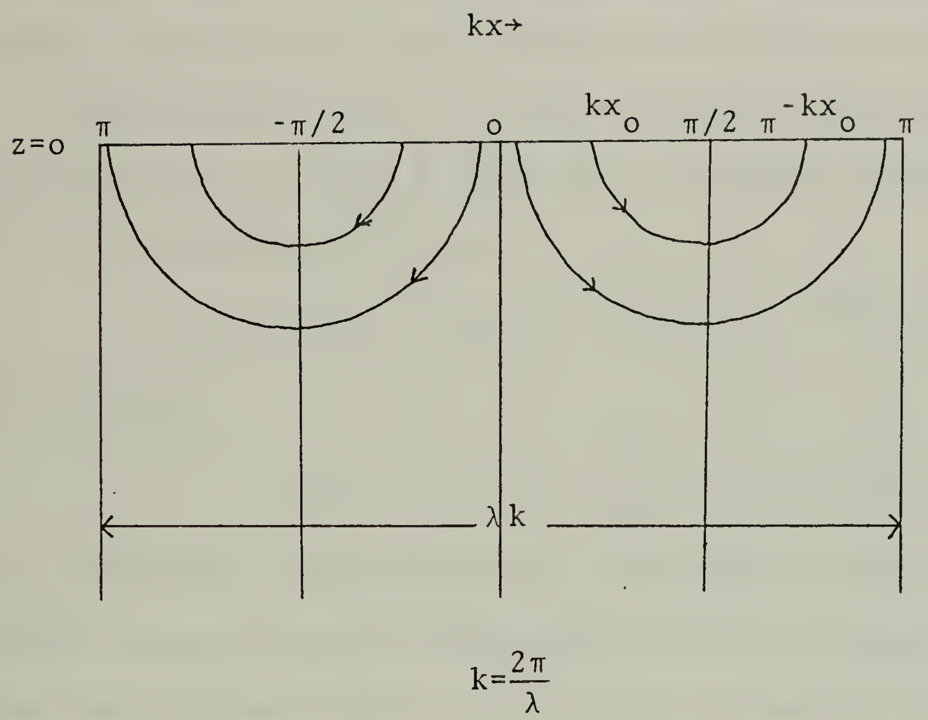

Figure 3

Cell Structure of the Circulation Field Model

IV. SOLUTION TO TRANSPORT EQUATION

In this section, a look is taken at the bubble transport equation with the model that was presented in the previous section. The main interest will focus at determining the bubble distribution as a function of depth and bubble size.

Substituting into the bubble transport equation the expressions for $\bar{a}$ and R from the preceeding section results in the following equation:

$$\bar{v}\cdot\nabla_r\Psi + \bar{a}\cdot\nabla_v\Psi + R\partial\Psi/\partial l = \left[{}^{3\alpha}/l^2 - {}^{v_z}/3(\alpha_1 - z)\right]\Psi$$

By defining

$$\phi(z,x,v_z,v_x,l) \equiv \iint_{-\infty}^{\infty} \Psi(\bar{r},\bar{v},l)\,dy\,dv_y$$

and using the two-dimensional circulation field, the above equation can be integrated over the y and v_y directions and the problem reduces to solving the partial differential equation:

$$v_x\frac{\partial\phi}{\partial x} + v_z\frac{\partial\phi}{\partial z} + a_x\frac{\partial\phi}{\partial v_x} + a_z\frac{\partial\phi}{\partial v_z} + R\frac{\partial\phi}{\partial l} = \left[\frac{2\alpha}{l^2} - \frac{v_z}{3(\alpha_1 - z)}\right]\phi \quad (7)$$

with the boundary condition $\phi(0,x,v_x,v_z,l) = \phi_o\ (0,x_o,v_{zo},v_{xo},l_o)$ corresponding to a surface source.

Equation (7) is a first order, linear partial differential equation in 5 variables and can be solved by the method of characteristics (Ref. 9). Using this method, the partial differential equation is equivalent to the following 5 characteristic ordinary differential equations in terms of the parameter z, which was chosen for mathematical convience:

Characteristic Diff. Eqns. Characteristics

$$Z \le 0 \qquad\qquad z=0$$

$$\frac{dx}{dz} = \frac{v_x}{v_z} \qquad\qquad x=x_o$$

$$\frac{dv_x}{dz} = \frac{a_x}{v_z} \qquad\qquad v_x=v_{xo}$$

$$\frac{dv_z}{dz} = \frac{a_z}{v_z} \qquad\qquad v_z=v_{zo}$$

$$\frac{dl}{dz} = \frac{R}{v_l} \qquad\qquad l=l_o$$

$$\frac{d\phi}{dz} = \left[\frac{2\alpha}{l^2 v_z} - \frac{1}{3(\alpha_l - z)}\right]\phi \qquad\qquad \phi=\phi_o(x_o, v_{xo}, v_{zo}, l_o)$$

A close look at the characteristic ordinary differential equations reveals that they are simply the bubble dynamic equations.

Solving these equations for ϕ would be straight forward, except the equation for v_z is non-linear and analytic solutions have been found only for very special cases (Ref. 10). Thus, approximate methods must be considered to obtain v_z.

The method used for this paper to solve for v_z was to assume $v_z = V_z + u_z$. Appendix A outlines this method with the solution for u_z as:

$$u_z = e^{-\int_o^z \frac{\alpha}{l^2 v_z} dz'} \left[\int_o^z e^{\int_o^{z'} \frac{\alpha}{l^2 v_z} dz''} \left[\frac{g}{\sigma v_z} + \frac{1}{\sigma}\left(\frac{V_x}{V_z}\frac{\partial V_z}{\partial x} + \frac{\partial V_z}{\partial z}\right)\right] dz' + u_{zo}\right].$$

Since the drag acceleration on the bubble is large, the bubbles will be entrained by the fluid within the first few centimeters of travel and u_z thus approaches a terminal

velocity. Referring to the computer output for u_z, shows that the assumption $V_z >> u_z$ can be made and also u_z is independent of V_{zo}, thus implying that v_z is independent of v_{zo}.

By a similiar approach, it is shown in Appendix A that $v_x \approx V_x$. Now, the characteristic equation involving x can be written as:

$$\frac{dx}{dz} \approx \frac{V_x}{V_z} ,$$

and using the chosen circulation field,

$$x \approx 1/k \sin^{-1}(e^{-kz}\sin kx_o).$$

This equation implies that the bubbles follow trajectories in the fluid and form streamlines as was indicated on figure 3. Thus choosing a x_o one can describe the path which the bubbles follow.

The solution of the characteristic equation for l is straight forward with

$$l=(1-z/\alpha_1)^{-1/3}\left[l_o + \frac{\delta rt}{\alpha}\int_o^z \frac{z'(1-z'/\alpha_1)^{-2/3}}{v_z} dz'\right] .$$

The second term in this equation is a result of gas diffusion and limits the bubble lifetime and hence the depth that surface generated bubbles can penetrate.

From the last characteristic equation, ϕ takes the general form:

$$\phi(z,x_o,v_{xo},v_{zo},l_o) = \phi_o(x_o,v_{xo},v_{zo},l_o) \left[(1-z/\alpha_1)^{1/3} \text{EXP} \int_o^z \frac{2\alpha}{l^2 v_z} dz'\right]$$

where ϕ as a function of (x,z,v_x,v_z,l) is obtained from the relationships between the characteristics and their variables. Assume a surface source ϕ_o that is separable,

$$\phi_o(x_o,v_{xo},v_{zo},l_o) = X(x_o)Y_x(v_{xo})Y(v_{zo})L(l_o),$$

and for convience, let

$$Y_x(v_{xo}) = \delta(v_{xo}-\tilde{v}_x) \quad ; \quad Y_z(v_{zo}) = \delta(v_{zo}-\tilde{v}_z).$$

Since v_x and v_z are nearly independent of v_{xo} and v_{zo}, this choice will not strongly affect the solution. Therefore,

$$\phi(x,z,v_x,v_z,l) = X_o(x_o)L(l_o)\delta(v_{xo}-\tilde{v}_x)\delta(v_{zo}-v_z) \left[(1-z/\alpha_1)^{1/3} \text{EXP} \int_o^z \frac{2\alpha}{l^2 v_z} dz'\right],$$

Now, define the "bubble density" as a function of position and size,

$$\Phi(x,z,l) \equiv \iint_{-\infty}^{\infty} \phi(x,z,v_x,v_z,l)dv_x dv_z$$

and integrate the expression for ϕ over v_x and v_z:

$$\Phi(x,z,l) = \iint_{-\infty}^{\infty} X(x_o)L(l_o)\delta(v_{xo}-\tilde{v}_x)\delta(v_{zo}-\tilde{v}_z) \left[(1-z/\alpha_1\}^{1/3} \text{EXP} \int_o^z \frac{2\alpha}{l^2 v_z} dz'\right] dv_x\, dv_z\ ,$$

To take advantage of the delta-functions, the variables of integration are changed, $dv_x dv_z = J dv_{xo}\, dv_{zo}$, where J is the Jocobian:

$$J \equiv \frac{\partial(v_x,v_z)}{\partial(v_{xo},v_{zo})}$$

The equation of interest now has the form:

$$\Phi(x,z,l) = X(x_o)L(l_o)(1-z/\alpha_1)^{1/3}\ \mathrm{EXP}\int_o^z \frac{2\alpha}{l^2 v_z}\,dz' J \quad (8)$$

Because of the number of mathematical steps involved in deriving the Jocobian, the development is found in Appendix B. The final approximate result is:

$$J \approx \frac{v_{zo}}{v_z}\, e^{-\int_o^z \frac{2\alpha}{l^2 v_z}\,dz'}\ \mathrm{EXP}\int_o^z -\xi dz'$$

where,

$$\xi = +\frac{2\alpha}{l^3 v_z}(v_z - V_z)\frac{\delta rt}{\alpha_1}\int_o^z \frac{z'(1-z'/\alpha_1)}{v_z{}^2}\,dz'$$

Thus, the equation for the distribution of bubbles with one initial surface velocity can be written as (Ref. 12):

$$\Phi(x,z,l) \approx X(x_o)L(l_o)(1-z/\alpha_1)^{1/3}\frac{v_{zo}}{v_z}\, e^{\int_o^z \xi dz'} \quad (9)$$

Looking at equation (9), the term $e^{\int_o^z \xi dz'}$ will be the predominate feature in solving for Φ. Two things should be noted about ξ. First, ξ is seen to be proportional to $1/V_o{}^3$ when the approximation $v_z \approx V_z$ is used. This result illustrates the importance of selecting a good circulation field model. Secondly, $\xi\alpha\delta RT$ and shows that gas diffusions is very important in the solution for the bubble distribution.

V. RESULTS

A computer program was written for calculating equation (9), to determine if the results were physically realistic. In this program, the assumptions were made that $X(x_o)=1$ and $L(l_o)=Kl_o^{-7/2}$ (Ref. 11). For the latter assumption, K=1 was used in the computions since the interest was in the shape of the distribution curve rather than the actual densities. Calculations for three circulation field velocities were made and the following graphs have been plotted:

Z_{max} vs. l_o

$\Phi\ (0,z,l)$ vs. Z for various radii

$\Phi\ (0,z,l)$ vs. l for various depths.

Z_{max} is the maximum penetration depth for a given size surface generated bubble. All calculations were made for $x_o=0$, which implies a straight line trajectory downward. If some other x_o had been chosen, line integration would had to be performed on the characteristic curves in 5-space (x,z,v_x,v_3,l).

The numerical technique used in computing the integrals was 10-point Gaussian Quadrature, which approximates the integrand by a Legendre polynomial series. Although this technique was fairly accurate for most of the integrals, it could not be used in computing the relative velocity (u_z). For this calculation, it is noted that the integrand is essentially zero except for the last few percent of the

interval (0,z) where it increases dramatically. U_z was therefore evaluated by using double precision 32-point Gaussian Quadrature on the latter portion of (0,z).

In computing the relative velocity, the approximation

$$\int_{o}^{z} \frac{\alpha}{l^2 v_z}\, dz' \approx \frac{\alpha}{l^2 V_o}\, z e^{-kz}\ , \ x=0,$$

was used which is accurate provided $|Z| < 10$ meters. Also, since the calculations involved taking the difference between two such integrals, very little error was introduced.

It is appropriate now to make a few comments about the results that have been plotted. Figure 4 is a graph of maximun depth versus inital bubble size. This graph shows that bubbles have a finite life time as a result of gas diffusion and that smaller bubbles introduced at the surface will not penetrate as deep as larger bubbles.

Figures 5 through 7 are plots of Φ (0,z,1) versus depth for various bubble radii. These graphs show the influence that the circulation field velocity has on how deep the bubbles will penetrate. It should also be noted that even though the surface density falls off as $l_o^{-3.5}$, there is a certain depth where the larger bubbles become more populous than the smaller bubbles because of gas diffusion.

Figures 8 through 11 are plots of Φ (0,z,1) versus radius for various depths. The main feature about these graphs is that the initial distribution appears to persist to some depth which is a function of the circulation field velocity.

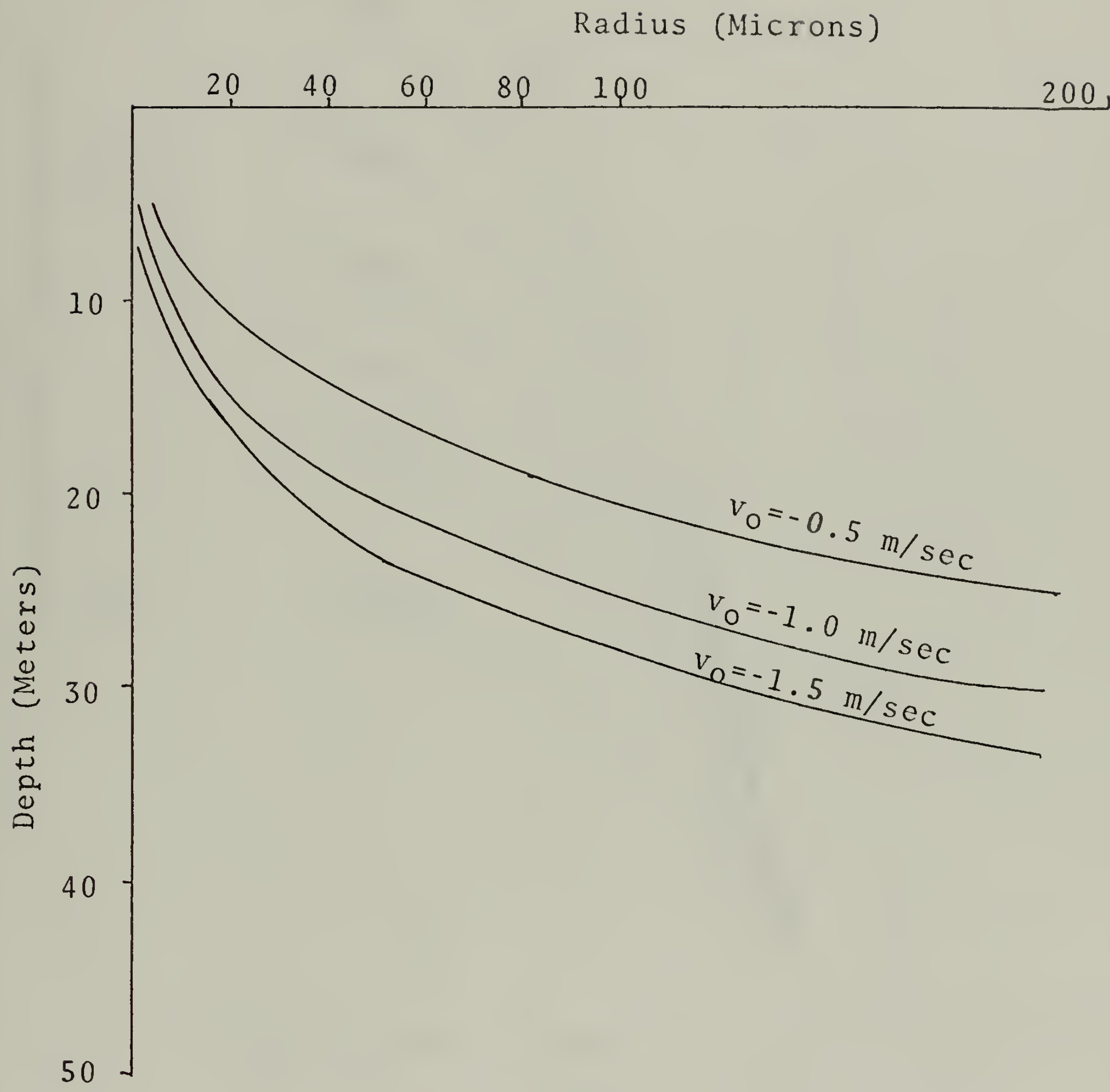

Figure 4

Curves for Maximun Penetration vs. Bubble Radius for Various Circulation Field Velocities

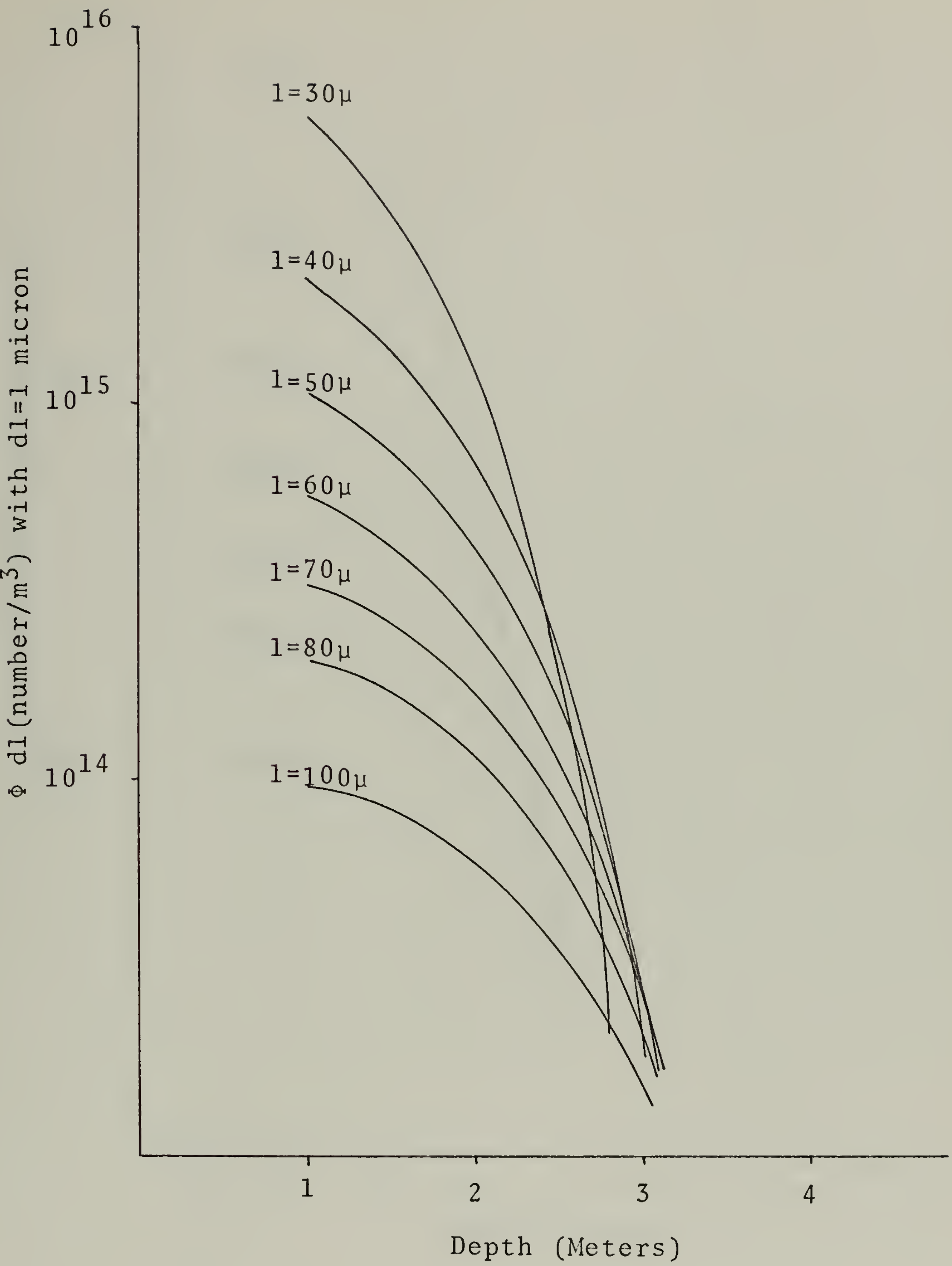

Figure 5

Bubble Density vs Depth for Various Size Bubbles, v_o=-0.5 m/sec.

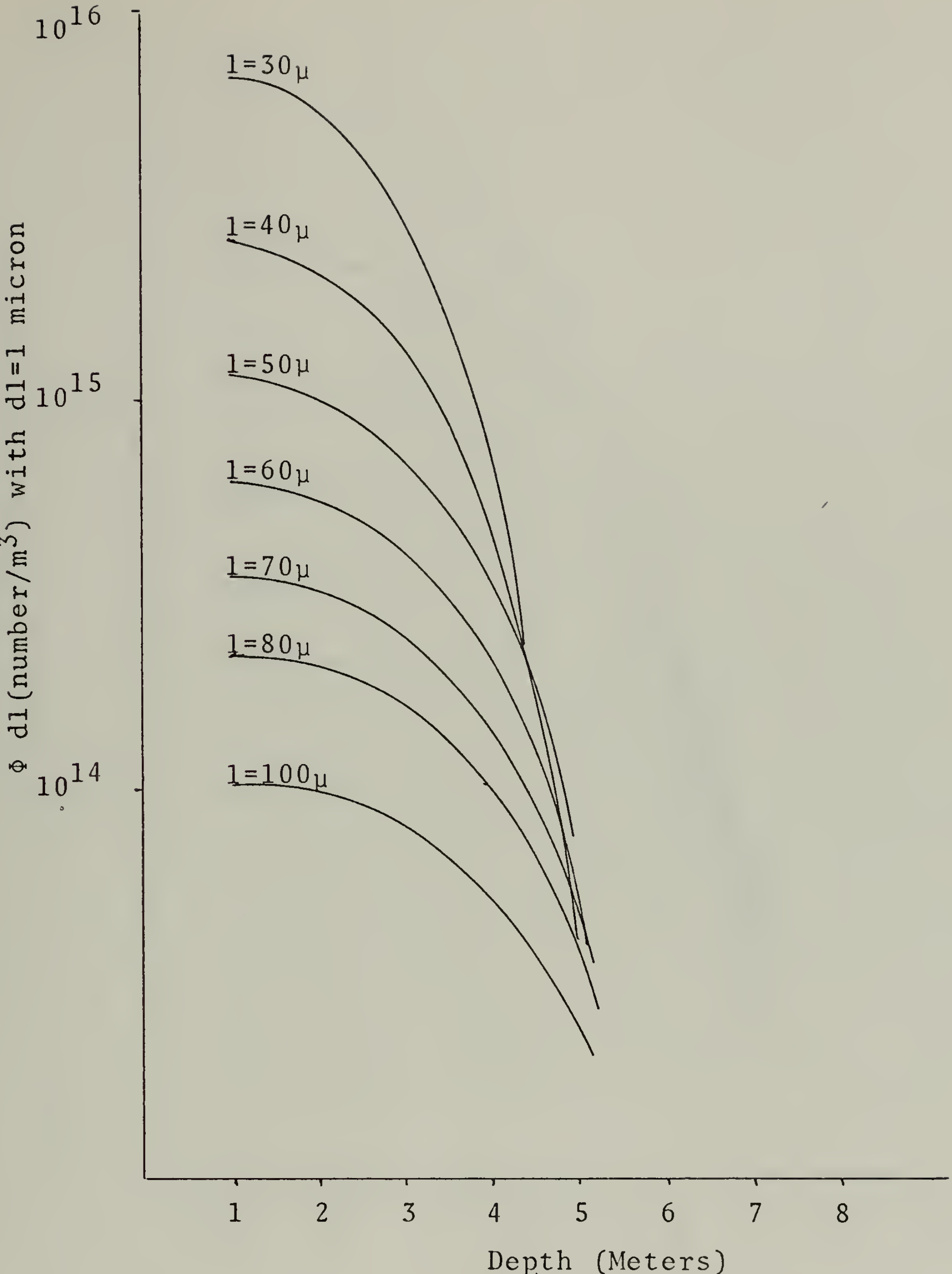

Figure 6

Bubble Density vs Depth for Various Size Bubbles, v_o=-1.0 m/sec.

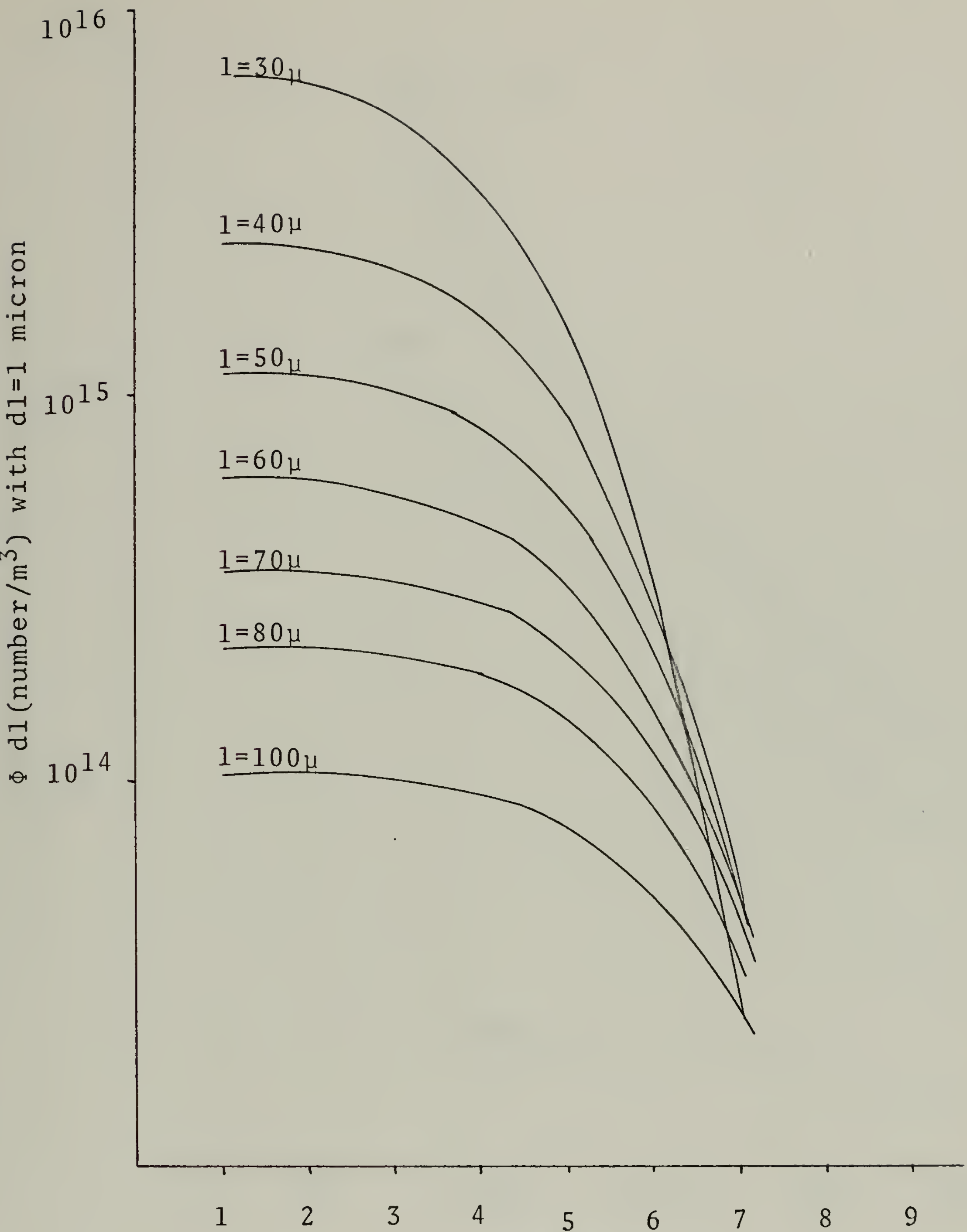

Figure 7

Bubble Density vs. Depth for Various Size Bubbles, v_o=-1.5 m/sec.

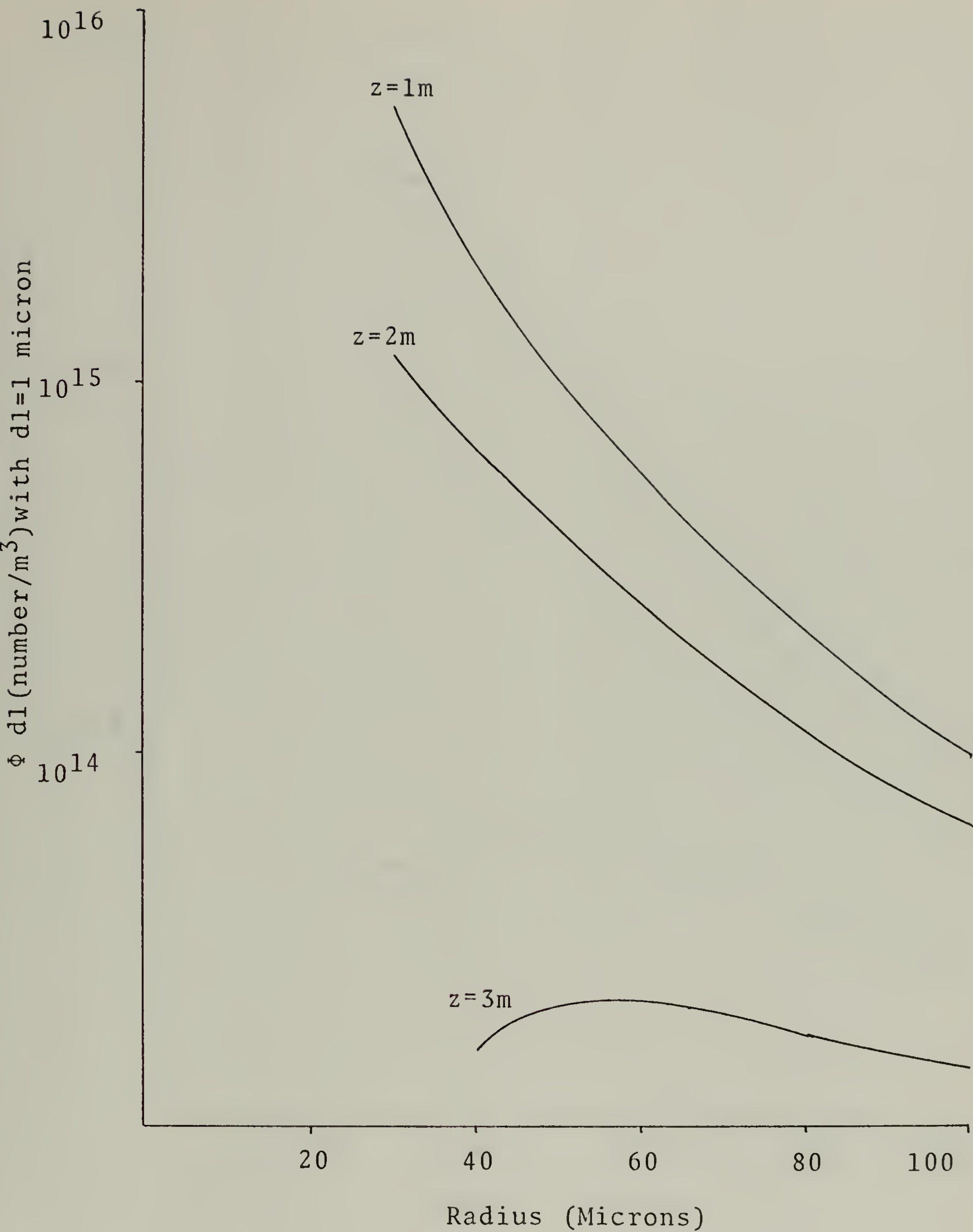

Figure 8

Bubble Density vs Bubble Radius
for Various Depths, v_o=-0.5 m/sec.

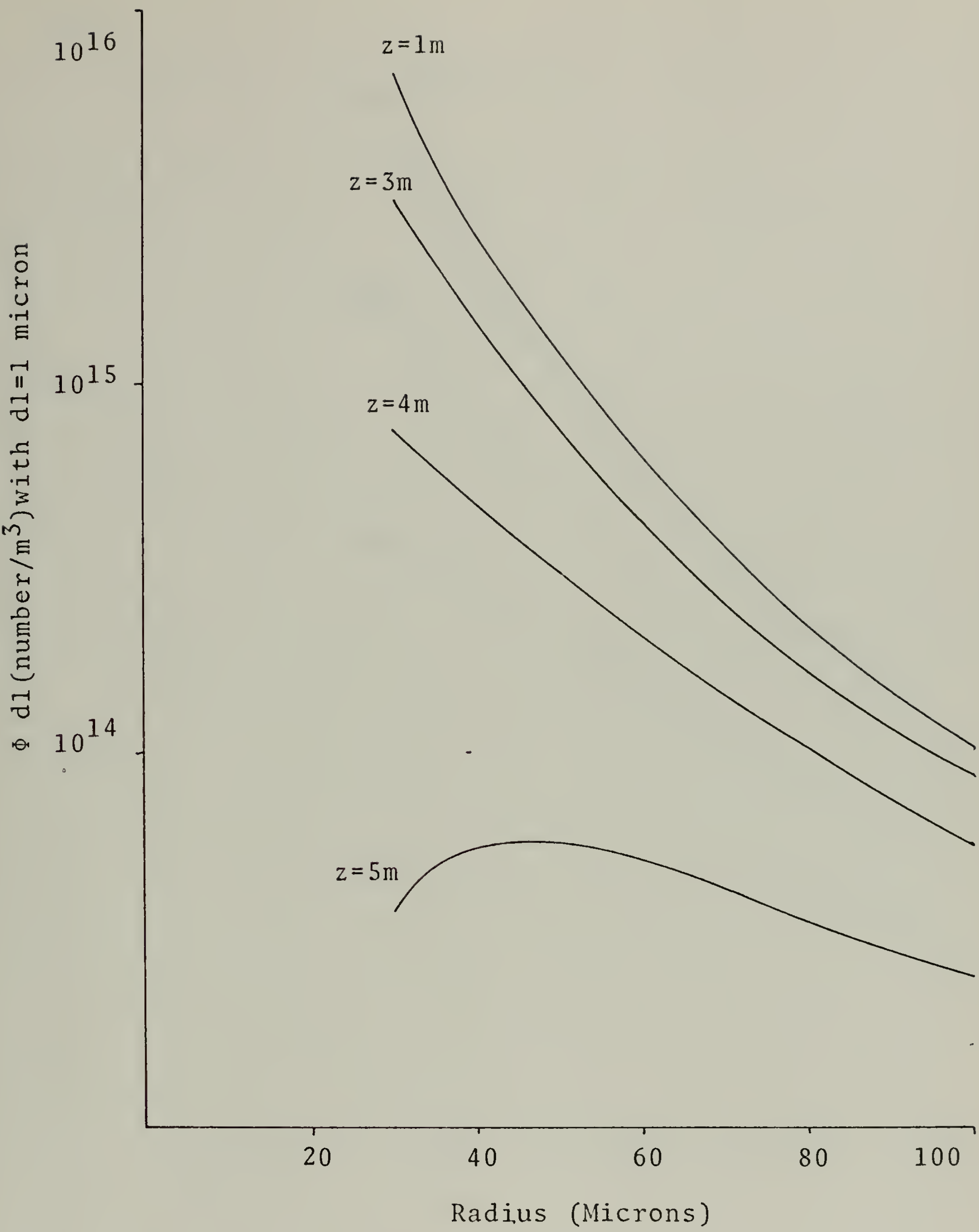

Figure 9

Bubble Density vs Bubble Radius
for Various Depths, v_o=-1.0 m/sec.

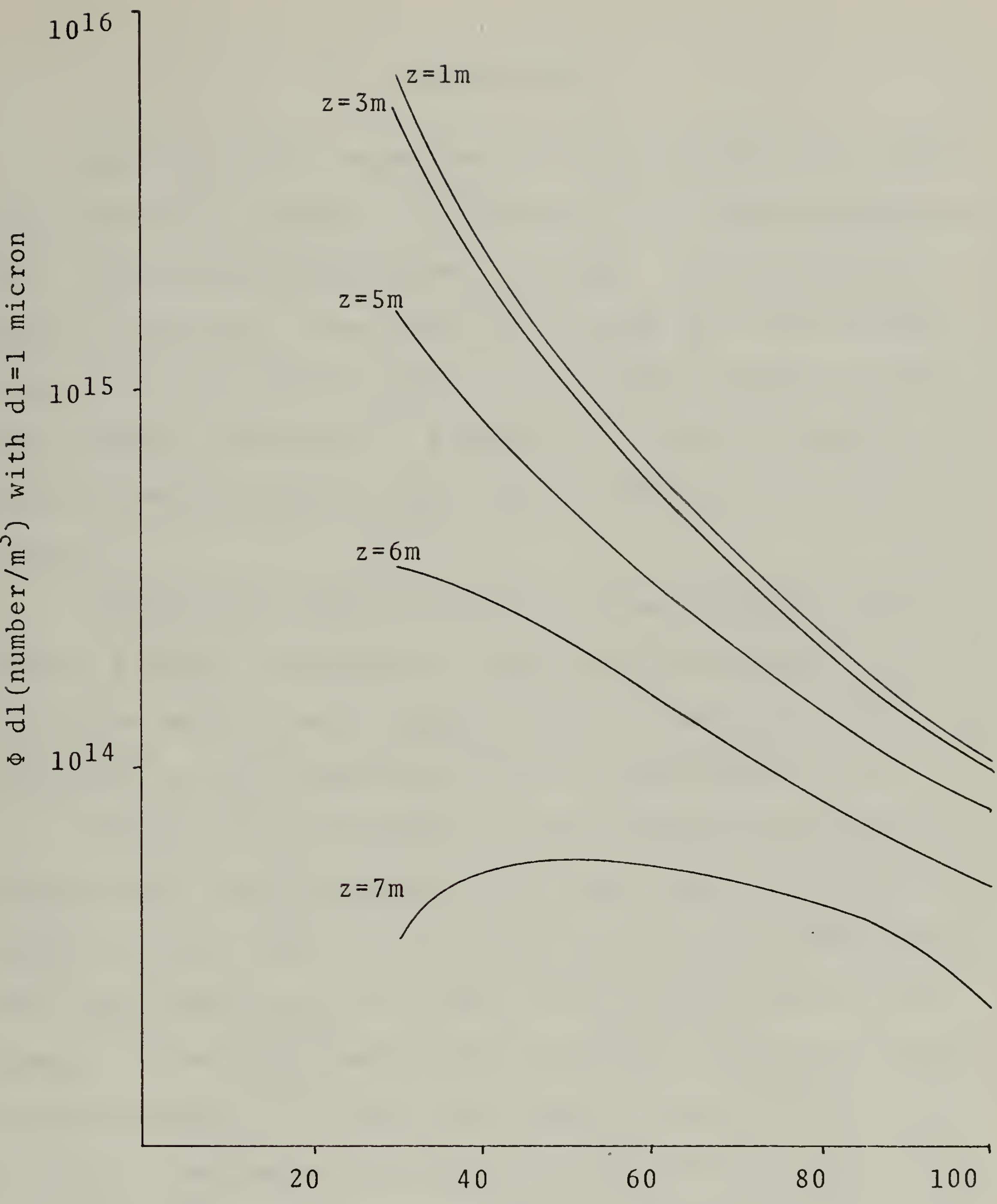

Figure 10

Bubble Density vs Bubble Radius
for Various Depths, v_o=-1.5 m/sec.

VI. CONCLUSIONS

There is little experimental data available to compare the results to, however, in reference (5) where measurements were taken acoustically, general shapes for bubble distributions are given. One should note that the experimental data in reference (5) is not for isoradius bubbles since the resonant frequency is a function of depth, however, a good comparision can be made within a few meters of the surface.

Although the bubble distribution as computed does not follow a simple mathematical function, the results for the first few meters can be approximated by exponental laws, depending on the circulation velocity and bubble size.

Looking first at figure 5, the distribution drops off rapidly with depth pointing to the fact that for small circulation velocities, a distributive source is needed since the depth penetration is small. For V_o=-1.0 m/sec, the curves can be approximated for the first 3 meters by the exponental laws, $e^{-z/3}$ for small bubbles and $e^{-z/9}$ for large bubbles. The curves for V_o=-1.5 m/sec follow fairly close $e^{-z/9}$ for the smaller bubbles when $z \geq -3.5$m and $e^{-z/15}$ for larger bubbles when $z \geq -4.5$m. The curves for V_o=-1.0 and V_o=-1.5m /sec compare favorably with the experimental data mentioned, especially for the smaller bubbles.

Even though there is fairly good agreement in the distribution near the surface, a definite discrepancy does

occur at the deeper depths. Probably, the major reason for this discrepancy is the fact that no distributive or bottom sources were considered in the calculations. The data in reference (5) was taken in shallow water and calm seas, so inclusion of such sources into the model should improve the agreement between the solution and the experimental data. Another possible reason for the discrepancy is that a large portion of the Jacobian was assumed small and neglected. This term was not looked at in detail and could be of some importance at the greater depths.

Since the results do agree in part with experimental data, the approach taken in this paper does appear feasible for determining the bubble distribution in the upper ocean. For further studies in this area, it is suggested that solutions to the bubble transport equation be extended to include calculations for $x \neq 0$, models with distributed sources and surface tension and better models for the circulation field and gas diffusion.

APPENDIX A

Solutions for the Relative Velocities

In this appendix, solutions are obtained for the relative velocities from the characteristic equations. Consider first the characteristic equation for v_z, which is non-linear. Write $v_z = V_z + u_z$, differentiate:

$$\frac{dv_z}{dz} = \frac{dV_z}{dz} + \frac{du_z}{dz}$$

Substituting this result into the characteristic equation for v_z gives

$$\frac{du_z}{dz} + \frac{\alpha}{1^2}\frac{u_z}{v_z} = \frac{g}{\sigma v_z} + \frac{1}{\sigma}\left[\frac{v_x}{v_z}\frac{\partial V_z}{\partial_x} + \frac{\partial V_z}{\partial z}\right]$$

A formal solution to this equation can be written:

$$u_z = e^{-\int_0^z \frac{\alpha}{1^2 v_z} dz'}\left[\int_0^z e^{\int_0^{z'} \frac{\alpha}{1^2 v_z{}^2} dz''}\left[\frac{g}{\sigma v_z} + \frac{1}{\sigma}\left(\frac{v_x}{v_z}\frac{\partial v_z}{\partial x} + \frac{\partial v_z}{\partial z}\right)\right]\right.$$

$$\left. dz' + u_{zo}\right],$$

or, rewritting,

$$u_z = \int_0^z e^{\int_z^{z'} \frac{\alpha}{1^2 v_z} dz''}\left[\frac{g}{\sigma v_z} + \frac{1}{\sigma}\left(\frac{v_x}{v_z}\frac{\partial v_z}{\partial x} + \frac{\partial v_z}{\partial z}\right)\right] dz' + u_{zo} e^{-\int_0^z \frac{\alpha}{1^2 v_z} dz'}$$

For computational purposes, the approximation $v_z \cong V_z$ will be made where appropriate, since the above expression is an implicit equation. This approximation turns out to be valid as indicated in the Results Section.

In order to solve the characteristic equation for v_x, the same approach is used as described above, with the result:

$$u_x = e^{-\int_0^z \left[\frac{\alpha}{2v_z} - \frac{1}{\sigma v_z}\frac{\partial v_x}{\partial x}\right] dz'} \left[\int_0^z e^{\int_0^{z'} \left[\frac{\alpha}{2v_z} - \frac{1}{\sigma v_z}\frac{\partial V_x}{\partial x}\right] dz''} \left[\frac{1}{\sigma}\left(\frac{\partial V_x}{\partial z} + \frac{v_x}{v_z}\frac{\partial V_x}{\partial x}\right)\right] dz' + u_{xo}\right].$$

A close look at this equation reveals that u_x is approximately zero since the exponental term shows a sharp decaying feature and the circulation field chosen does not change rapidly. Thus the approximation $v_x \tilde{=} V_x$ can be used in the calculations.

APPENDIX B

Development of the Jacobian

This section is devoted to the mathematical manipulations involved in developing an expression for the Jacobian required for equation (8). The following are the functional relationships among the variables and characteristics that need to be considered:

$$v_x = v_x(z, x_o, v_{xo}, v_{zo}, l_o)$$
$$v_z = v_z(z, x_o, v_{xo}, v_{zo}, l_o)$$
$$l = l\ (z, x_o, v_{xo}, v_{zo}, l_o)$$
$$x = x\ (z, x_o, v_{xo}, v_{zo}, l_o)$$
$$V_x = V_x(z, x_o, v_{xo}, v_{zo}, l_o, V_{xo})$$
$$V_z = V_z(z, x_o, v_{xo}, v_{zo}, l_o, V_{zo}).$$

For mathematical convience, define:

$$J_1 \equiv \partial v_x/\partial v_{xo} \quad ; \quad J_2 \equiv \partial v_x/\partial v_{zo} \quad ; \quad J_3 \equiv \partial v_z/\partial v_{xo}$$
$$J_4 \equiv \partial v_z/\partial v_{zo} \quad ; \quad J_5 \equiv \partial x/\partial v_{xo} \quad ; \quad J_6 \equiv \partial x/\partial v_{zo}.$$

Thus,

$$J = \frac{\partial(v_x, v_z)}{\partial(v_{xo}, v_{zo})} = J_1J_4 - J_2J_3.$$

Now, using the characteristic equations for v_x, v_z and x and noting that, for example:

$$\frac{\partial}{\partial v_{xo}}\left(\frac{dv_x}{dz}\right) = \frac{dJ_1}{dz} ,$$

the following equations can be written:

$$\frac{dJ_1}{dz} = \left[-\frac{\alpha}{l^2 v_z} - \frac{1+\sigma}{\sigma}\frac{k\,V_z}{v_z}\right] J_1 + \left[\frac{\alpha}{l^2}\frac{(v_x - V_x)}{v_z 2} + \frac{1+\sigma}{\sigma}\frac{k\,V_z}{v_z 2} v_x\right] J_3$$

$$+ \frac{\alpha}{l^2 v_z}\frac{\partial V_x}{\partial v_{xo}} - \frac{1+\sigma}{\sigma}\frac{v_x}{v_z} k \frac{\partial V_z}{\partial v_{xo}} + \frac{1+\sigma}{\sigma} k \frac{\partial V_x}{\partial v_{xo}} + \frac{2\alpha}{l^3 v_z}(v_x - V_x)\frac{\partial l}{\partial v_{xo}}$$

$$\frac{dJ_2}{dz} = \left[-\frac{\alpha}{l^2 v_z} - \frac{1+\sigma}{\sigma}\frac{k\,V_z}{v_z}\right] J_2 + \left[\frac{\alpha}{l^2}\frac{(v_x - V_x)}{v_z 2} + \frac{1+\sigma}{\sigma}\frac{v_x}{v_z 2} k\, V_z\right] J_4$$

$$+ \frac{\alpha}{l^2 v_z}\frac{\partial V_x}{\partial v_{zo}} - \frac{1+\sigma}{\sigma}\frac{v_x}{v_z} k \frac{\partial V_z}{\partial v_{zo}} + \frac{1+\sigma}{\sigma} k \frac{\partial V_x}{\partial v_{zo}} + \frac{2\alpha}{l^3 v_z}(v_x - V_x)\frac{\partial l}{\partial v_{zo}}$$

$$\frac{dJ_3}{dz} = \left[-\frac{\alpha}{l^2 v_z 2} V_z \quad \frac{-g}{\sigma v_z 2} - \frac{1+\sigma}{\sigma}\frac{v_x}{v_z 2} kV_x\right] J_3 + \left[\frac{1+\sigma}{\sigma}\frac{k\,V_x}{v_z}\right] J_1$$

$$+ \frac{\alpha}{l^2 v_z}\frac{\partial V_z}{\partial v_{xo}} + \frac{1+\sigma}{\sigma}\frac{v_x}{v_z} k \frac{\partial V_x}{\partial v_{xo}} + \frac{1+\sigma}{\sigma} k \frac{\partial V_z}{\partial v_{xo}} + \frac{2\alpha}{l^3 v_z}(v_z - V_z)\frac{\partial l}{\partial v_{xo}}$$

$$\frac{dJ_4}{dz} = \left[-\frac{\alpha}{l^2}\frac{V_z}{v_z 2} - \frac{g}{\sigma v_z 2} - \frac{1+\sigma}{\sigma}\frac{v_x}{v_z 2} kV_x\right] J_4 + \left[\frac{1+\sigma}{\sigma}\frac{k\,V_x}{v_z}\right] J_2$$

$$+ \frac{\alpha}{l^2 v_z}\frac{\partial V_z}{\partial v_{zo}} + \frac{1+\sigma}{\sigma}\frac{v_x}{v_z} k \frac{\partial V_x}{\partial v_{zo}} + \frac{1+\sigma}{\sigma} k \frac{\partial V_z}{\partial v_{zo}} + \frac{2\alpha}{l^3 v_z}(v_z - V_z)\frac{\partial l}{\partial v_{zo}}$$

For ease of manipulation, define:

$$A \equiv -\frac{\alpha}{l^2 v_z} - \frac{1+\sigma}{\sigma}\frac{kV_z}{v_z} \quad ; \quad B \equiv \frac{\alpha}{l^2}\frac{(v_x - V_x)}{v_z 2} + \frac{1+\sigma}{\sigma} kV_z \frac{v_x}{v_z 2}$$

$$C \equiv -\frac{\alpha}{l^2 v_z 2} V_z - \frac{g}{\sigma v_z 2} - \frac{1+\sigma}{\sigma}\frac{kV_x v_x}{v_z 2} \quad ; \quad D \equiv \frac{1+\sigma}{\sigma}\frac{kV_x}{v_x}$$

$$E \equiv -\frac{\alpha}{l^2 v_z} kV_z - \frac{1+\sigma}{\sigma}\frac{v_x}{v_z} k^2 V_x - \frac{1+\sigma}{\sigma} k^2 V_z$$

$$F \equiv -\frac{\alpha}{l^2 v_z} kV_x - \frac{1+\sigma}{\sigma}\frac{v_x}{v_z} kV_z + \frac{1+\sigma}{\sigma} k^2 V_x$$

$$q_1 \equiv \frac{2\alpha}{1^3 v_z}(v_x - V_x) \quad ; \quad q_2 \equiv \frac{2\alpha}{1^3 v_z}(v_z - V_z)$$

The six equations can now be written as:

$$\frac{dJ_1}{dz} = AJ_1 + BJ_3 + E\,J_5 + q_1 \frac{\partial 1}{\partial v_{xo}}$$

$$\frac{dJ_2}{dz} = AJ_2 + BJ_4 + E\,J_6 + q_1 \frac{\partial 1}{\partial v_{zo}}$$

$$\frac{dJ_3}{dz} = CJ_3 + DJ_1 + F\,J_5 + q_2 \frac{\partial 1}{\partial v_{xo}}$$

$$\frac{dJ_4}{dz} = CJ_4 + DJ_2 + F\,J_6 + q_2 \frac{\partial 1}{\partial v_{zo}}$$

$$\frac{dJ_5}{dz} = \frac{1}{v_z} J_1 - \frac{v_x}{v_z 2} J_3$$

$$\frac{dJ_6}{dz} = \frac{1}{v_z} J_2 - \frac{x}{v_z 2} J_4 \quad .$$

Taking the moments of the first four equations:

$$J_4 \frac{dJ_1}{dz} = AJ_1J_4 + BJ_3J_4 + EJ_4J_5 + q_1\frac{\partial 1}{\partial v_{xo}} J_4 = \frac{d}{dz}(J_1J_4) - J_1\frac{dJ_4}{dz}$$

$$J_3 \frac{dJ_2}{dz} = AJ_2J_3 + BJ_3J_4 + EJ_3J_6 + q_1\frac{\partial 1}{\partial v_{zo}} J_3 = \frac{d}{dz}(J_2J_3) - J_2\frac{dJ_3}{dz}$$

$$J_2 \frac{dJ_3}{dz} = CJ_2J_3 + DJ_1J_2 + FJ_2J_5 + q_2\frac{\partial 1}{\partial v_{xo}} J_2 = \frac{d}{dz}(J_2J_3) - J_3\frac{dJ_2}{dz}$$

$$J_1 \frac{dJ_4}{dz} = CJ_1J_4 + DJ_1J_2 + FJ_1J_6 + q_1\frac{\partial l}{\partial v_{zo}} J_1 = \frac{d}{dz}(J_1J_4) - J_4\frac{dJ_4}{dz}$$

Now, combining the above equations give the result:

$$\frac{dJ}{dz} = (A+C)J + q_1\left[\frac{\partial l}{\partial v_{xo}} J_4 - \frac{\partial l}{\partial v_{zo}} J_3\right] + q_2\left[\frac{\partial l}{\partial v_{zo}} J_1 - \frac{\partial l}{\partial v_{xo}} J_2\right] + Q$$

where

$$Q \equiv E(J_4J_5 - J_3J_6) + F(J_1J_6 - J_2J_5).$$

Using the equation for l that was developed earlier and taking the partial derivatives gives:

$$\frac{\partial l}{\partial v_{xo}} = -\left[\frac{\delta rt}{\alpha_1}\int_o^z \frac{z'(1-z'/\alpha_1)^{-2/3}}{v_z^2} J_3\, dz'\right](1-z/\alpha_1)^{-1/3}$$

and

$$\frac{\partial l}{\partial v_{zo}} = -\left[\frac{\delta rt}{\alpha_1}\int_o^z \frac{z'(1-z'/\alpha_1)^{-2/3}}{v_z^2} J_4\, dz'\right](1-z/\alpha_1)^{-1/3}.$$

If gas diffusion had been neglected in the development, the above two equations would have been zero and the solution to the transport equation would become unphysical.

Substituting the above results into the differential equation for J results in:

$$\frac{dJ}{dz} = (A+C-\xi)J + q_1\left[\frac{\delta rt}{\alpha_1}\int_o^z \frac{z'(1-z'/\alpha_1)^{-2/3}}{v_z^2}(J_3J_4' - J_3'J_4)\,dz'\right]$$

$$-q_2\left[\frac{\delta rt}{\alpha_1}\int_o^z \frac{z'(1-z'/\alpha_1)^{-2/3}}{v_z^2}\left[(J_1J_4' - J_2J_3') - (J_1'J_4 - J_2'J_3)\right] dz'\right]$$

$$+Q$$

where now,

$$\xi \equiv q_2 \frac{\delta r t}{\alpha_1} \int_o^z \frac{z'(1-z'/\alpha_1)^{-2/3} dz'}{v_z^2} .$$

For simplification and as a first approximation assume:

$$\frac{dJ}{dz} \approx (A+C-\xi)J$$

and the solution to this differential equation is:

$$J = \mathrm{EXP} \int_o^z (A+C-\xi)\, dz'$$

since J = 1 at z=o.

```
THE INPUT DATA IS ALPH,ALPH1,XKAP,VZO,SIGMA,GRAV,RTDEL RESPECTIVELY
   0.875E-05   0.100E 02   0.100E 00  -0.500E 00   0.500E 00   0.980E 01   0.100E-05

AND THE VERTICAL ARGUMENTS Z ARE
     1.000     1.500     2.000     2.500     3.000     3.500     4.000
     4.500     5.000     5.500     6.000     6.500     7.000     8.000
     9.000    10.000

THE BUBBLE RADII ARE
 0.300E-04 0.400E-04 0.500E-04 0.600E-04 0.700E-04 0.800E-04 0.100E-03
```

****THE SURFACE DICTRIBUTION IS CAPX=1 AND CAPL=LO**-3.5

THE PROBABILITY DENSITY----FOR THE BUBBLE RADII,L(MICRONS)

Z	L= 30.	L= 40.	L= 50.	L= 60.	L= 70.	L= 80.	L= 100.
-1.00	0.563E 16	0.216E 16	0.102E 16	0.551E 15	0.326E 15	0.207E 15	0.960E 14
-1.50	0.342E 16	0.149E 16	0.762E 15	0.432E 15	0.265E 15	0.173E 15	0.835E 14
-2.00	0.118E 16	0.678E 15	0.407E 15	0.257E 15	0.170E 15	0.117E 15	0.615E 14
-2.50	0.174E 15	0.163E 15	0.131E 15	0.101E 15	0.765E 14	0.585E 14	0.353E 14
-3.00	0.755E 13	0.159E 14	0.206E 14	0.217E 14	0.207E 14	0.187E 14	0.143E 14
-3.50	0.604E 11	0.446E 12	0.121E 13	0.207E 13	0.279E 13	0.326E 13	0.356E 13
-4.00	0.492E 08	0.235E 10	0.190E 11	0.667E 11	0.149E 12	0.254E 12	0.469E 12
-4.50	0.192E 04	0.135E 07	0.525E 08	0.513E 09	0.236E 10	0.689E 10	0.267E 11
-5.00	0.138E-02	0.434E 02	0.151E 05	0.620E 06	0.779E 07	0.478E 08	0.520E 09
-5.50	0.541E-11	0.332E-04	0.239E 00	0.694E 02	0.344E 04	0.582E 05	0.256E 07
-6.00	0.252E-22	0.210E-12	0.915E-07	0.375E-03	0.118E 00	0.777E 01	0.222E 04
-6.50	0.200E-37	0.289E-23	0.311E-15	0.438E-10	0.160E-06	0.643E-04	0.217E 00
-7.00	0.241E-57	0.164E-37	0.271E-26	0.407E-19	0.378E-14	0.162E-10	0.139E-05
-8.00	0.0	0.0	0.457E-59	0.940E-46	0.146E-36	0.743E-30	0.955E-21
-9.00	0.0	0.0	0.0	0.0	0.0	0.0	0.478E-45
-10.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0

RELZ AS A FUNCTION OF Z(METERS) AND L(MICRONS)

Z	L= 30.	L= 40.	L= 50.	L= 60.	L= 70.	L= 80.	L= 100.
-1.00	0.180E-02	0.321E-02	0.501E-02	0.722E-02	0.982E-02	0.128E-01	0.200E-01
-1.50	0.173E-02	0.307E-02	0.479E-02	0.690E-02	0.940E-02	0.123E-01	0.192E-01
-2.00	0.166E-02	0.294E-02	0.460E-02	0.662E-02	0.901E-02	0.118E-01	0.184E-01
-2.50	0.159E-02	0.282E-02	0.441E-02	0.635E-02	0.865E-02	0.113E-01	0.176E-01
-3.00	0.153E-02	0.272E-02	0.424E-02	0.611E-02	0.832E-02	0.109E-01	0.170E-01
-3.50	0.147E-02	0.262E-02	0.409E-02	0.588E-02	0.801E-02	0.105E-01	0.163E-01
-4.00	0.142E-02	0.252E-02	0.394E-02	0.568E-02	0.772E-02	0.101E-01	0.158E-01
-4.50	0.137E-02	0.244E-02	0.381E-02	0.548E-02	0.746E-02	0.974E-02	0.152E-01
-5.00	0.133E-02	0.236E-02	0.368E-02	0.530E-02	0.721E-02	0.942E-02	0.147E-01
-5.50	0.128E-02	0.228E-02	0.356E-02	0.513E-02	0.698E-02	0.912E-02	0.142E-01
-6.00	0.124E-02	0.221E-02	0.345E-02	0.497E-02	0.676E-02	0.883E-02	0.138E-01
-6.50	0.121E-02	0.214E-02	0.335E-02	0.482E-02	0.656E-02	0.857E-02	0.134E-01
-7.00	0.117E-02	0.208E-02	0.325E-02	0.468E-02	0.637E-02	0.831E-02	0.130E-01
-8.00	0.110E-02	0.196E-02	0.307E-02	0.442E-02	0.601E-02	0.785E-02	0.123E-01
-9.00	0.105E-02	0.186E-02	0.291E-02	0.418E-02	0.570E-02	0.744E-02	0.116E-01
-10.00	0.994E-03	0.177E-02	0.276E-02	0.398E-02	0.541E-02	0.707E-02	0.110E-01

```
THE INPUT DATA IS ALPH,ALPH1,XKAP,VZ0,SIGMA,GRAV,RTDEL RESPECTIVELY
    0.875E-05   0.100E 02   0.100E 00  -0.100E 01   0.500E 00   0.980E 01   0.100E-05

 AND THE VERTICAL ARGUMENTS Z ARE
       1.000      1.500      2.000      2.500      3.000      3.500      4.000
       4.500      5.000      5.500      6.000      6.500      7.000      8.000
       9.000     10.000
 THE BUBBLE RADII ARE
  0.300E-04 0.400E-04 0.500E-04 0.600E-04 0.700E-04 0.800E-04 0.100E-03
```

****THE SURFACE DICTRIBUTION IS CAPX=1 AND CAPL=LO**-3.5

THE PROBABILITY DENSITY----FOR THE BUBBLE RADII,L(MICRONS)

Z	L= 30.	L= 40.	L= 50.	L= 60.	L= 70.	L= 80.	L= 100.
-1.00	0.670E 16	0.247E 16	0.113E 16	0.601E 15	0.351E 15	0.220E 15	0.101E 15
-1.50	0.634E 16	0.237E 16	0.110E 16	0.588E 15	0.345E 15	0.218E 15	0.100E 15
-2.00	0.559E 16	0.217E 16	0.103E 16	0.557E 15	0.330E 15	0.210E 15	0.977E 14
-2.50	0.443E 16	0.183E 16	0.902E 15	0.500E 15	0.302E 15	0.194E 15	0.922E 14
-3.00	0.303E 16	0.138E 16	0.724E 15	0.418E 15	0.259E 15	0.170E 15	0.834E 14
-3.50	0.169E 16	0.899E 15	0.515E 15	0.316E 15	0.205E 15	0.139E 15	0.711E 14
-4.00	0.716E 15	0.477E 15	0.312E 15	0.209E 15	0.144E 15	0.103E 15	0.560E 14
-4.50	0.212E 15	0.195E 15	0.154E 15	0.116E 15	0.878E 14	0.667E 14	0.399E 14
-5.00	0.396E 14	0.564E 14	0.578E 14	0.519E 14	0.442E 14	0.367E 14	0.249E 14
-5.50	0.407E 13	0.106E 14	0.154E 14	0.174E 14	0.175E 14	0.164E 14	0.132E 14
-6.00	0.196E 12	0.114E 13	0.266E 13	0.410E 13	0.510E 13	0.563E 13	0.565E 13
-6.50	0.362E 10	0.614E 11	0.266E 12	0.615E 12	0.102E 13	0.139E 13	0.187E 13
-7.00	0.200E 08	0.138E 10	0.135E 11	0.528E 11	0.127E 12	0.227E 12	0.446E 12
-8.00	0.436E 01	0.203E 05	0.220E 07	0.410E 08	0.292E 09	0.117E 10	0.697E 10
-9.00	0.648E-10	0.329E-03	0.187E 01	0.442E 03	0.184E 05	0.269E 06	0.946E 07
-10.00	0.340E-27	0.160E-15	0.541E-09	0.741E-05	0.510E-02	0.581E 00	0.335E 03

RELZ AS A FUNCTION OF Z(METERS) AND L(MICRONS)

Z	L= 30.	L= 40.	L= 50.	L= 60.	L= 70.	L= 80.	L= 100.
-1.00	0.179E-02	0.319E-02	0.498E-02	0.717E-02	0.976E-02	0.127E-01	0.199E-01
-1.50	0.172E-02	0.305E-02	0.477E-02	0.686E-02	0.934E-02	0.122E-01	0.191E-01
-2.00	0.165E-02	0.293E-02	0.457E-02	0.658E-02	0.896E-02	0.117E-01	0.183E-01
-2.50	0.158E-02	0.281E-02	0.439E-02	0.632E-02	0.861E-02	0.112E-01	0.176E-01
-3.00	0.152E-02	0.270E-02	0.423E-02	0.608E-02	0.828E-02	0.108E-01	0.169E-01
-3.50	0.147E-02	0.261E-02	0.407E-02	0.586E-02	0.798E-02	0.104E-01	0.163E-01
-4.00	0.141E-02	0.251E-02	0.393E-02	0.566E-02	0.770E-02	0.101E-01	0.157E-01
-4.50	0.137E-02	0.243E-02	0.379E-02	0.546E-02	0.744E-02	0.971E-02	0.152E-01
-5.00	0.132E-02	0.235E-02	0.367E-02	0.528E-02	0.719E-02	0.939E-02	0.147E-01
-5.50	0.128E-02	0.227E-02	0.355E-02	0.511E-02	0.696E-02	0.909E-02	0.142E-01
-6.00	0.124E-02	0.220E-02	0.344E-02	0.496E-02	0.675E-02	0.881E-02	0.138E-01
-6.50	0.120E-02	0.214E-02	0.334E-02	0.481E-02	0.654E-02	0.855E-02	0.134E-01
-7.00	0.117E-02	0.207E-02	0.324E-02	0.467E-02	0.635E-02	0.830E-02	0.130E-01
-8.00	0.110E-02	0.196E-02	0.306E-02	0.441E-02	0.600E-02	0.784E-02	0.123E-01
-9.00	0.105E-02	0.186E-02	0.290E-02	0.418E-02	0.569E-02	0.743E-02	0.116E-01
-10.00	0.993E-03	0.177E-02	0.276E-02	0.397E-02	0.541E-02	0.706E-02	0.110E-01

```
THE INPUT DATA IS ALPH,ALPH1,XKAP,VZO,SIGMA,GRAV,RTDEL RESPECTIVELY
   0.875E-05   0.100E 02   0.100E 00  -0.150E 01   0.500E 00   0.980E 01   0.100E-05

 AND THE VERTICAL ARGUMENTS Z ARE
      1.000      1.500      2.000      2.500      3.000      3.500      4.000
      4.500      5.000      5.500      6.000      6.500      7.000      8.000
      9.000     10.000
 THE BUBBLE RADII ARE
  0.300E-04 0.400E-04 0.500E-04 0.600E-04 0.700E-04 0.800E-04 0.100E-03
```

****THE SURFACE DICTRIBUTION IS CAPX=1 AND CAPL=LO**-3.5

THE PROBABILITY DENSITY----FOR THE BUBBLE RADII,L(MICRONS)

Z	L= 30.	L= 40.	L= 50.	L= 60.	L= 70.	L= 80.	L= 100.
-1.00	0.683E 16	0.250E 16	0.115E 16	0.607E 15	0.354E 15	0.222E 15	0.102E 15
-1.50	0.676E 16	0.249E 16	0.115E 16	0.608E 15	0.355E 15	0.223E 15	0.102E 15
-2.00	0.656E 16	0.244E 16	0.113E 16	0.603E 15	0.354E 15	0.222E 15	0.102E 15
-2.50	0.617E 16	0.234E 16	0.110E 16	0.590E 15	0.348E 15	0.220E 15	0.102E 15
-3.00	0.555E 16	0.218E 16	0.104E 16	0.564E 15	0.336E 15	0.214E 15	0.998E 14
-3.50	0.471E 16	0.193E 16	0.949E 15	0.525E 15	0.316E 15	0.203E 15	0.963E 14
-4.00	0.369E 16	0.162E 16	0.827E 15	0.470E 15	0.289E 15	0.188E 15	0.909E 14
-4.50	0.261E 16	0.126E 16	0.678E 15	0.400E 15	0.252E 15	0.167E 15	0.832E 14
-5.00	0.161E 16	0.882E 15	0.515E 15	0.319E 15	0.208E 15	0.142E 15	0.734E 14
-5.50	0.838E 15	0.547E 15	0.353E 15	0.234E 15	0.161E 15	0.114E 15	0.616E 14
-6.00	0.351E 15	0.289E 15	0.214E 15	0.155E 15	0.113E 15	0.841E 14	0.487E 14
-6.50	0.112E 15	0.125E 15	0.111E 15	0.903E 14	0.717E 14	0.566E 14	0.357E 14
-7.00	0.255E 14	0.422E 14	0.471E 14	0.448E 14	0.395E 14	0.338E 14	0.238E 14
-8.00	0.333E 12	0.177E 13	0.390E 13	0.577E 13	0.696E 13	0.749E 13	0.727E 13
-9.00	0.310E 09	0.111E 11	0.731E 11	0.220E 12	0.437E 12	0.680E 12	0.110E 13
-10.00	0.518E 04	0.393E 07	0.147E 09	0.136E 10	0.591E 10	0.164E 11	0.583E 11

RELZ AS A FUNCTION OF Z(METERS) AND L(MICRONS)

Z	L= 30.	L= 40.	L= 50.	L= 60.	L= 70.	L= 80.	L= 100.
-1.00	0.177E-02	0.315E-02	0.493E-02	0.710E-02	0.966E-02	0.126E-01	0.197E-01
-1.50	0.170E-02	0.302E-02	0.472E-02	0.680E-02	0.925E-02	0.121E-01	0.189E-01
-2.00	0.163E-02	0.290E-02	0.453E-02	0.653E-02	0.888E-02	0.116E-01	0.181E-01
-2.50	0.157E-02	0.279E-02	0.436E-02	0.627E-02	0.854E-02	0.112E-01	0.174E-01
-3.00	0.151E-02	0.269E-02	0.420E-02	0.604E-02	0.822E-02	0.107E-01	0.168E-01
-3.50	0.146E-02	0.259E-02	0.405E-02	0.583E-02	0.793E-02	0.104E-01	0.162E-01
-4.00	0.141E-02	0.250E-02	0.391E-02	0.562E-02	0.765E-02	0.100E-01	0.156E-01
-4.50	0.136E-02	0.242E-02	0.377E-02	0.543E-02	0.740E-02	0.966E-02	0.151E-01
-5.00	0.131E-02	0.234E-02	0.365E-02	0.526E-02	0.716E-02	0.935E-02	0.146E-01
-5.50	0.127E-02	0.226E-02	0.354E-02	0.509E-02	0.693E-02	0.905E-02	0.141E-01
-6.00	0.123E-02	0.219E-02	0.343E-02	0.494E-02	0.672E-02	0.878E-02	0.137E-01
-6.50	0.120E-02	0.213E-02	0.333E-02	0.479E-02	0.652E-02	0.852E-02	0.133E-01
-7.00	0.116E-02	0.207E-02	0.323E-02	0.465E-02	0.633E-02	0.827E-02	0.129E-01
-8.00	0.110E-02	0.195E-02	0.305E-02	0.440E-02	0.599E-02	0.782E-02	0.122E-01
-9.00	0.104E-02	0.185E-02	0.290E-02	0.417E-02	0.568E-02	0.741E-02	0.116E-01
-10.00	0.991E-03	0.176E-02	0.275E-02	0.396E-02	0.540E-02	0.705E-02	0.110E-01

```
THE INPUT DATA IS ALPH1,VZO,XKAP,DELRT RESPECTIVELY
    0.100E 02  -0.500E 00   0.100E 00   0.100E-05

 AND THE VERTICAL ARGUMENT Z IS

      5.000    10.000    15.000    20.000    20.000    25.000    30.000
    35.000    40.000    45.000    50.000
```

Z	GAS
-0.500E 01	-0.289E-05
-0.100E 02	-0.141E-04
-0.150E 02	-0.400E-04
-0.200E 02	-0.921E-04
-0.200E 02	-0.921E-04
-0.250E 02	-0.191E-03
-0.300E 02	-0.371E-03
-0.350E 02	-0.695E-03
-0.400E 02	-0.127E-02
-0.450E 02	-0.227E-02
-0.500E 02	-0.400E-02

```
THE INPUT DATA IS ALPH1,VZO,XKAP,DELRT RESPECTIVELY
    0.100E 02  -0.100E 01   0.100E 00   0.100E-05

 AND THE VERTICAL ARGUMENT Z IS
      5.000     10.000     15.000     20.000     20.000     25.000     30.000
    35.000     40.000     45.000     50.000
```

Z	GAS
-0.500E 01	-0.145E-05
-0.100E 02	-0.704E-05
-0.150E 02	-0.200E-04
-0.200E 02	-0.460E-04
-0.200E 02	-0.460E-04
-0.250E 02	-0.954E-04
-0.300E 02	-0.186E-03
-0.350E 02	-0.348E-03
-0.400E 02	-0.633E-03
-0.450E 02	-0.113E-02
-0.500E 02	-0.200E-02

```
THE INPUT DATA IS ALPH1,VZO,XKAP,DELRT RESPECTIVELY
    0.100E 02  -0.150E 01   0.100E 00   0.100E-05

 AND THE VERTICAL ARGUMENT Z IS
      5.000    10.000    15.000    20.000    20.000    25.000    30.000
     35.000    40.000    45.000    50.000
```

Z	GAS
-0.500E 01	-0.964E-06
-0.100E 02	-0.469E-05
-0.150E 02	-0.133E-04
-0.200E 02	-0.307E-04
-0.200E 02	-0.307E-04
-0.250E 02	-0.636E-04
-0.300E 02	-0.124E-03
-0.350E 02	-0.232E-03
-0.400E 02	-0.422E-03
-0.450E 02	-0.755E-03
-0.500E 02	-0.133E-02

```
      **THIS PROGRAM IS TO COMPUTE THE DISTRIBUTION OF BUBBLES AS A
      **FUNCTION OF Z, FOR VARIOUS RADII'S.
      **MAIN PROGRAM
      DIMENSION ZZ(20),ARG1(20),FINT1(20),ARG2(20),FINT2(20),STO(20,20),
     1RL(20),RAD(20),STL(20,20)
      COMMON ALPH,ALPH1,XLO,XKAP,VZO,SIGMA,GRAV,RTDEL
      READ(5,1)NUMZ,NUML
      READ(5,3)(ZZ(I),I=1,NUMZ)
      READ(5,2)(RL(I),I=1,NUML)
 2100 READ(5,2,END=2101)ALPH,ALPH1,XKAP,VZO,SIGMA,GRAV,RTDEL
      WRITE(6,5)
      WRITE(6,6)ALPH,ALPH1,XKAP,VZO,SIGMA,GRAV,RTDEL,(ZZ(J),J=1,NUMZ)
      WRITE(6,15)(RL(J),J=1,NUML)
      **ZZ IS THE ABSOLUTE VALUE OF Z.
    1 FORMAT(8I10)
    2 FORMAT(8E10.3)
    3 FORMAT(8F10.3)
      DO 1000 IL=1,NUML
      XL=RL(IL)
      RAD(IL)=XL*1.0E 06
      DO 1000 I=1,NUMZ
      Z=-ZZ(I)
      CALL GASZ(Z,GAS)
      XLO=(XL*(1-Z/ALPH1)**.3333)-(RTDEL/(ALPH1*VZO))*GAS
      CALL TAB(0.0,Z,ARG1)
      ZI=0.0
      R1=0.0
      DO 1001 J=1,10
      CALL TAB(ZI,ARG1(J),ARG2)
      DO 1002 K=1,10
      FINT1(K)=(ARG2(K)*EXP(-2.0*XKAP*ARG2(K)))/(1-ARG2(K)/ALPH1)**.6667
 1002 CONTINUE
      W=ARG1(J)-ZI
      CALL GAUS10(FINT1,W,WRK)
      R1=R1+WRK
      ZI=ARG1(J)
      CALL EPSZ(ARG1(J),EPS)
      CALL GASZ(ARG1(J),GAS)
      XXL=(XLO+(RTDEL/(ALPH1*VZO))*GAS)/(1-ARG1(J)/ALPH1)**.33333
      FINT2(J)=R1*EXP(-XKAP*ARG1(J))*EPS/(XXL*XXL*XXL)
 1001 CONTINUE
      SAVE=XLO
      XLO=XL
      CALL EPSZ(Z.EPS)
      STL(I,IL)=EPS
      XLO=SAVE
      CALL GAUS10(FINT2,Z,R2)
      R=(-2.0*ALPH*RTDEL*R2)/(VZO*VZO*VZO*ALPH1)
```

```
      IF(R.LT.-174.) GO TO 3000
      GO TO 3100
 3000 XI=0
      GO TO 3200
 3100 XI=EXP(R)
 3200 ANS=XI*EXP(-XKAP*Z)*(1-Z/ALPH1)**.33333/(XLO**3.5)
      STO(I,IL)=ANS
    5 FORMAT('1')
    6 FORMAT('0','THE INPUT DATA IS ALPH,ALPH1,XKAP,VZO,SIGMA,GRAV,RTDEL
     1,RESPECTIVELY'/'0',7E12.3//'0','AND THE VERTICAL ARGUMENTS ZZ(J) A
     2RE'/'0',(12F10.3))
   15 FORMAT('0','THE BUBBLE RADII ARE'/'0',(10E10.3))
 1000 CONTINUE
      WRITE(6,16)
   16 FORMAT('1','**** THE SURFACE DISTRIBUTION IS CAPX=1 AND CAPL=LO**-
     13.5')
      WRITE(6,13)(RAD(J),J=1,NUML)
      DO 1010 I=1,NUMZ
      Z=-ZZ(I)
      WRITE(6,14)Z,(STO(I,J),J=1,NUML)
 1010 CONTINUE
   13 FORMAT('0','THE PROBABILITY DENSITY----FOR THE BUBBLE RADII,L(MICR
     1ONS)'/'0','    Z    ',10('    L=    ',F5.0))
      WRITE(6,17)(RAD(J),J=1,NUML)
      DO 1011 I=1,NUMZ
      Z=-ZZ(I)
      WRITE(6,14)Z,(STL(I,J),J=1,NUML)
 1011 CONTINUE
   17 FORMAT('1'/'0','RELZ AS A FUNCTION OF Z(METERS) AND L(MICRONS)'/'0
     1','    Z    ',10('    L=    ',F5.0))
   14 FORMAT('0',F6.2,10E12.3)
      GO TO 2100
 2101 CONTINUE
      STOP
      END


      FUNCTION F3(Z)
      COMMON ALPH,ALPH1,XLO,XKAP,VZO,SIGMA,GRAV,RTDEL
      F3=Z*EXP(-XKAP*Z)/((1-Z/ALPH1)**.66667)
      RETURN
      END
      **SUBROUTINE GAUS10 WITH SUBROUTINE TAB IS USED TO EVALUATE
      **THE DESIRED INTEGRATION. 10-POINT GAUSS QUADRATURE IS THE
      **TECHNIQUE USED.
```

```
      SUBROUTINE GAUS10(FINT,Z,R)
      DIMENSION W(5),FINT(10)
      R=0
      W(1)=.03333567
      W(2)=.07472567
      W(3)=.1095432
      W(4)=.1346334
      W(5)=.1477621
**THESE WEIGHTING FUNCTIONS CAME FROM SUBROUTINE QG10
      DO 101 J=1,5
      R=R+W(J)*(FINT(J)+FINT(11-J))
  101 CONTINUE
      R=Z*R
      RETURN
      END
**SUBROUTINE TAB IS TO TABULATE THE ARGUMENTS OF FINT
**IN SUBROUTINE GAUS10


      SUBROUTINE TAB(ZL,ZU,ARG)
      DIMENSION G(5),ARG(10)
      A=.5*(ZL+ZU)
      B=(ZU-ZL)
      G(1)=.4869533
      G(2)=.4325317
      G(3)=.3397048
      G(4)=.2166977
      G(5)=.07443717
      DO 101 J=1,5
      ARG(J)=A-B*G(J)
      ARG(11-J)=A+B*G(J)
  101 CONTINUE
      RETURN
      END
**SUBROUTINE EPSZ IS TO COMPUTE THE RELATIVE VEL IN THE Z DIRECTION


      SUBROUTINE EPSZ(Z,EPS)
      IMPLICIT REAL*8(D)
      COMMON ALPH,ALPH1,XLO,XKAP,VZO,SIGMA,GRAV,RTDEL
      DIMENSION DARG(40),DFINT(40)
      DR(DZP,DZ)=-GAM*(DZP*DEXP(-XKAP*DZP)-DZ*DEXP(-XKAP*DZ))
**THIS APPROXIMATION TO THE INTEGRAL,GAM*Z*EXP(-K*Z),SHOULD BE
**VERY ACCURATE SINCE WE ARE TAKING THE DIFFERENCE BETWEEN TWO OF
**THE INTEGRALS EVALUATED AT POINTS THAT ARE VERY CLOSE
**(LIKE ZP-ZPP=-10**-3)
      GAM=-ALPH/(XLO*XLO*VZO)
      DZ=Z
```

```
      DEL=-.100D-03
      DSTEP=DZ
   61 DSTEP=DSTEP-DEL
      DR1=DR(DSTEP,DZ)
      IF(DR1.GT.-.200D 02) GO TO 61
      CALL DTAB(DSTEP,DZ,DARG)
      DO 101 J=1,32
      DR1=DR(DARG(J),DZ)
      DFINT(J)=DEXP(DR1)*((GRAV/VZO)*DEXP(-XKAP*DARG(J))-XKAP*VZO*DEXP(X
     1KAP*DARG(J)))/SIGMA
  101 CONTINUE
      DW=DZ-DSTEP
      CALL DG32(DFINT,DW,DEPS)
      EPS=DEPS
      RETURN
      END


      SUBROUTINE DG32(DFINT,DZ,DRE)
      IMPLICIT REAL*8(D)
      **HERE IS THE LENGTH OF THE INTERVAL,DZ=DZU-DZL
      DIMENSION DW(16),DFINT(32)
      DRE=0
      DW(1)=.35093050047350483D-2
      DW(2)=.81371973654528350D-2
      DW(3)=.12696032654631030D-1
      DW(4)=.17136931456510717D-1
      DW(5)=.21417949011113340D-1
      DW(6)=.25499029631188088D-1
      DW(7)=.29342046739267774D-1
      DW(8)=.32911111388180923D-1
      DW(9)=.36172897054424253D-1
      DW(10)=.39096947893535153D-1
      DW(11)=.41655962113473378D-1
      DW(12)=.43826046502201906D-1
      DW(13)=.45586939347881942D-1
      DW(14)=.46922199540402283D-1
      DW(15)=.47819360039637430D-1
      DW(16)=.48270044257363900D-1
      DO 101 J=1,16
      DRE=DRE+DW(J)*(DFINT(J)+DFINT(33-J))
  101 CONTINUE
      DRE=DZ*DRE
      RETURN
      END
```

```
      SUBROUTINE DTAB(DZL,DZU,DARG)
      IMPLICIT REAL*8(D)
      DIMENSION DG(16),DARG(32)
      DA=.5D0*(DZL+DZU)
      DB=(DZU-DZL)
      DG(1)=.49863193092474078D0
      DG(2)=.49280575577263417D0
      DG(3)=.48238112779375322D0
      DG(4)=.46745303796886984D0
      DG(5)=.44816057788302606D0
      DG(6)=.42468380686628499D0
      DG(7)=.38724189798397120D0
      DG(8)=.36609105937014484D0
      DG(9)=.33152213346510760D0
      DG(10)=.29385787862038116D0
      DG(11)=.25344995446611470D0
      DG(12)=.21067563806531767D0
      DG(13)=.16593430114106382D0
      DG(14)=.11964368112606854D0
      DG(15)=.72235980791398250D-1
      DG(16)=.24153832843869158D-1
      DO 101 J=1,16
      DARG(J)=DA-DB*DG(J)
      DARG(33-J)=DA+DB*DG(J)
  101 CONTINUE
      RETURN
      END

      SUBROUTINE GASZ(Z,GAS)
      COMMON ALPH,ALPH1,XLO,XKAP,VZO,SIGMA,GRAV,RTDEL
      EXTERNAL F3
      CALL QG10(0.0,Z,F3,GAS)
      RETURN
      END

      SUBROUTINE QG10(XL,XU,FCT,Y)                                      QG10 360
      A=.5*(XU+XL)                                                      QG10 390
      B=XU-XL                                                           QG10 400
      C=.4869533*B                                                      QG10 410
      Y=.03333567*(FCT(A+C)+FCT(A-C))                                   QG10 420
      C=.4325317*B                                                      QG10 430
      Y=Y+.07472567*(FCT(A+C)+FCT(A-C))                                 QG10 440
      C=.3397048*B                                                      QG10 450
      Y=Y+.1095432*(FCT(A+C)+FCT(A-C))                                  QG10 460
      C=.2166977*B                                                      QG10 470
      Y=Y+.1346334*(FCT(A+C)+FCT(A-C))                                  QG10 480
```

```
      C=.07443717*B                                                     QG10 490
      Y=B*(Y+.1477621*(FCT(A+C)+FCT(A-C)))                              QG10 500
      RETURN                                                            QG10 510
      END                                                               QG10 520
```

```
      **THIS IS A PROGRAM TO COMPUTE THE INTEGRAL OF
      **DELRT*Z*EXP(-KZ)/A1*VZO(1-Z/A1)2/3
      DIMENSION ZZ(100)
      COMMON ALPH1,VZO,XKAP,DELRT
      EXTERNAL F3
      READ(5,2)NUM
      READ(5,3)(ZZ(I),I=1,NUM)
 2100 READ(5,1,END=2000)ALPH1,VZO,XKAP,DELRT
      WRITE(6,8)
      WRITE(6,4)ALPH1,VZO,XKAP,DELRT,(ZZ(J),J=1,NUM)
    1 FORMAT(4E10.3)
    2 FORMAT(8I10)
    3 FORMAT(8F10.3)
    4 FORMAT('0','THE INPUT DATA IS ALPH1,VZO,XKAP,DELRT RESPECTIVELY'/'
     10',4E12.3//'0','AND THE VERTICAL ARGUMENT Z IS'/'0',(12F10.3))
      WRITE(6,7)
      DO 1000 I=1,NUM
      Z=-ZZ(I)
      CALL QG10(0.0,Z,F3,R)
      WRITE(6,6)
      WRITE(6,5)Z,R
    5 FORMAT('0',2E12.3)
    6 FORMAT('0')
    7 FORMAT('1','      Z              GAS       ')
    8 FORMAT('1')
 1000 CONTINUE
      GO TO 2100
 2000 CONTINUE
      STOP
      END


      FUNCTION F3(Z)
      COMMON ALPH1,VZO,XKAP,DELRT
      F3=(DELRT/(ALPH1*VZO))*Z*EXP(-XKAP*Z)/((1-Z/ALPH1)**.66667)
      RETURN
      END


      SUBROUTINE QG10(XL,XU,FCT,Y)
      A=.5*(XU+XL)
      B=XU-XL
      C=.4869533*B
      Y=.03333567*(FCT(A+C)+FCT(A-C))
      C=.4325317*B
      Y=Y+.07472567*(FCT(A+C)+FCT(A-C))
      C=.3397048*B
      Y=Y+.1095432*(FCT(A+C)+FCT(A-C))
```

```
C=.2166977*B
Y=Y+.1346334*(FCT(A+C)+FCT(A-C))
C=.07443717*B
Y=B*(Y+.1477621*(FCT(A+C)+FCT(A-C)))
RETURN
END
```

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13. ABSTRACT

A bubble transport equation has been developed that the bubble distribution, as a function of position, velocity, and size, must satisfy. An analytical model for bubble transport in the upper ocean is chosen and solutions are developed for this model. For a surface source only, these solutions compare favorably to experimental data in the near-surface region. The depth of this region of agreement depends upon the circulation field chosen, but is of the order of 3 meters.

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